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Observer-based feedback stabilization of linear systems with event-triggered sampling and dynamic quantization[☆]

Aneel Tanwani[a,*], Christophe Prieur[b], Mirko Fiacchini[b]

[a] LAAS (CNRS), 7 Avenue Colonel du Roche, 31400 Toulouse, France

[b] Gipsa-Lab (CNRS), 11 Rue des Mathématiques, BP 46, 38402 St. Martin d'Hères, France

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ABSTRACT

We consider the problem of output feedback stabilization in linear systems when the measured outputs and control inputs are subject to event-triggered sampling and dynamic quantization. A new sampling algorithm is proposed for outputs which does not lead to accumulation of sampling times and results in asymptotic stabilization of the system. The approach for output sampling is based on defining an event function that compares the difference between the current output and the most recently transmitted output sample not only with the current value of the output, but also takes into account a certain number of previously transmitted output samples. This allows us to reconstruct the state using an observer with sample-and-hold measurements. The estimated states are used to generate a control input, which is subjected to a different event-triggered sampling routine; hence the sampling times of inputs and outputs are asynchronous. Using Lyapunov-based approach, we prove the asymptotic stabilization of the closed-loop system and show that there exists a minimum inter-sampling time for control inputs and for outputs. To show that these sampling routines are robust with respect to transmission errors, only the quantized (in space) values of outputs and inputs are transmitted to the controller and the plant, respectively. A dynamic quantizer is adopted for this purpose, and an algorithm is proposed to update the range and the centre of the quantizer that results in an asymptotically stable closed-loop system.

1. Introduction

For sampled data control of continuous-time dynamical systems, event-triggered techniques have regained interest over the past 5–6 years [1], where the measurements are not sent periodically to the controller, but instead the sampling times are determined based on the current value of the state. A recent article [2] provides a tutorial exposition into the subject, and sums up most of the work done so far. A common framework for event-triggered control involves a stabilizing feedback controller and a triggering mechanism that determines when to send the updated measurements to the controller. While the feedback control is usually available "off-the-shelf", different strategies and variants are adopted for triggering mechanism depending upon the particular problem setup. Initial approaches for event-triggering mechanism involve keeping the difference between current value of the state and the last updated measurement relatively small [3–5]. Another technique is to monitor the derivative of the Lyapunov function associated with the closed-loop system, and if it starts approaching zero, then we update the measurement knowing that the new measurement will make the derivative sufficiently negative [6–8]. The effect of disturbances in plant dynamics could also be taken into account by such methods if the triggering mechanism is modified appropriately [9]. Moreover, event-triggered control has also been used for stabilization of systems in the presence of networks [10–13].

In the references cited above, the triggering mechanisms are based on using the full-state measurements and when it comes to using output (partial state) measurements for feedback, rather than full-state feedback, then relatively little has been done. If we directly generalize the techniques based on keeping the error (between the last sampled output and the current value of the output) small, then such methods lead to Zeno phenomenon, where we need to send infinitely many samples in finite-time and hence the technique is not feasible for implementation in

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* Corresponding author.
E-mail addresses: aneel.tanwani@laas.fr (A. Tanwani), christophe.prieur@gipsa-lab.fr (C. Prieur), miko.fiacchini@gipsa-lab.fr (M. Fiacchini).

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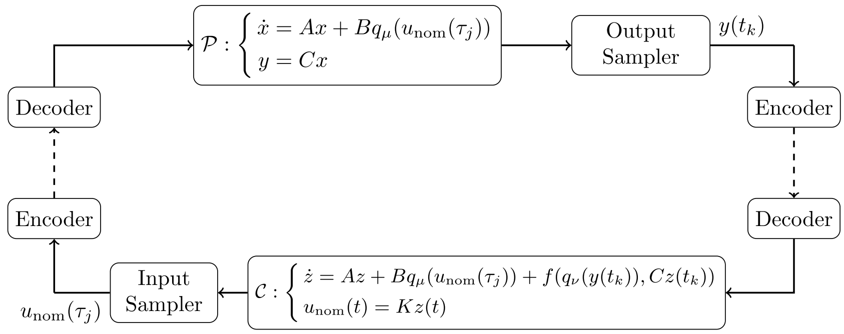

Fig. 1. Feedback loop where the inputs and outputs are time-sampled and quantized.

practice. Some refinements have been proposed in [14–16], where instead of asymptotic stabilization, the authors modify the event function to achieve practical stabilization so that the trajectories of the closed-loop system only converge to a ball defined as a design parameter. Eventually, that parameter also determines the minimum inter-sampling time as well. Asymptotic stability with output-feedback and event-triggered sampling has also been considered in more recent works where a certain dwell-time is enforced between two consecutive sample updates to overcome Zeno phenomenon. The so-called periodic event-triggered control could be seen as an implementation of this idea [17,18] where it is assumed that the continuous-time plant is already discretized with some fixed sampling-time, or a certain sampling period is precalculated to asymptotically stabilize the system. One then focuses on adding another level of sampling strategy (which is event-triggered) that would reduce the sampling rate for measurements even further. The results appearing in [18] for linear systems take disturbances into account, and derive minimum inter-sampling time for full-state feedback case only. The idea of forcing a certain dwell-time between two consecutive sample-updates has also been adopted in nonlinear setting for output feedback laws in [19]. Different from these techniques, the output sampling strategy in [20] proposes the use of Luenberger observer to estimate the state and use it in computation of sampling times.

In this paper, we propose a dynamic output feedback controller for asymptotic stabilization using event-triggered sampling which does not rely on precalculating some fixed sampling period between output updates. Our framework involves computing the sampling times for outputs and inputs separately. Just like the approach adopted in the state-feedback case, our approach is also based on keeping the error between the current value of the output and the last sampled output small. The crux of our approach is to compare this difference not only with the norm of the current value of the output, but with the norm of a vector that comprises some previously transmitted output measurements. If we pick a sufficient number of past samples, then these samples contain enough information about the norm of the state (due to the observability assumption). The controller, using these sampled outputs, is designed based on the principle of *certainty equivalence*. An estimate of the current state is first computed, which is in turn fed into the control law. The control inputs transmitted to the plant are also time-sampled, where the event-triggering rule depends upon the state of the controller. To show that our sampling algorithms are feasible for implementation, we derive an expression for minimum inter-sampling time between the sampled measurements sent to the controller and the plant. A property of the proposed sampling routines is that the sampling times of the output and control input are not necessarily synchronized.

As an added practical consideration and to show that our strategy is robust with respect to transmission errors, we assume that the sampled outputs and sampled inputs are subjected to quantization as well, that is, the output and inputs are transmitted to the controller and the plant, respectively using a string of finite alphabets only. However, to preserve asymptotic stability, the model of the quantizer is assumed to be dynamic as used in [21,22], that is the parameter that determines the range and the sensitivity of the quantizer can be scaled. Event-triggered sampling with static quantization is also considered in the works of [23] with full state feedback, and with output under a passivity assumption on the plant dynamics in [24], but none of these works allow the possibility of designing different sampling and quantization algorithms for inputs and outputs. The novelty in handling the quantization comes from the fact that we are working with an observer-based controller where the outputs and control inputs are both subject to quantization, and the sampling is event-based and not periodic.

To summarize, this paper proposes algorithms for event-triggered sampling and dynamic quantization of input and output measurements of linear time-invariant systems, also see Fig. 1. Such architectures could be useful when the control action is computed on a server located far away from the plant and the communication is carried over some communication channel between the plant and the controller. The paper proposes algorithm on how and when the information between the plant and the controller must be transmitted, and the contribution could be summed up through following observations:

- We can achieve asymptotic stabilization using dynamic output feedback and event-triggered sampling of the output measurements without imposing time-regularization or fixed periodic sampling as done in the literature, provided we use the information of previously sampled outputs, and not just the last sampled measurement.
- The event-triggered sampling algorithms are robust with respect to transmission errors, which in this paper manifest in the form of quantization. If these errors vanish (which happens due to dynamic quantization) then the state of the system also converges to the origin asymptotically.
- A trade-off between *how fast we sample* compared to *how precisely we quantize the measurements* also follows from our results. It appears in the form of design parameters introduced for sampling and quantization, respectively. In particular, when the dynamic parameter for quantization is very large (so that quantized measurements are very coarse), one has to sample quite fast, whereas smaller values of the quantization parameter (more exact measurements) possibly allow for larger inter-sampling times.

2. Problem setup

We consider linear time-invariant systems described as:

$$\mathcal{P}: \begin{cases} \dot{x}(t) = Ax(t) + Bu(t), \\ y(t) = Cx(t) \end{cases} \tag{1}$$