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Simultaneous interconnection and damping assignment passivity-based control of mechanical systems using dissipative forces

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ABSTRACT

To extend the realm of application of the well known controller design technique of interconnection and damping assignment passivity-based control (IDA-PBC) of *mechanical systems* two modifications to the standard method are presented in this article. First, similarly to Batlle et al. (2009) and Gómez-Estern and van der Schaft (2004), it is proposed to avoid the splitting of the control action into energy-shaping and damping injection terms, but instead to carry them out *simultaneously*. Second, motivated by Chang (2014), we propose to consider the inclusion of *dissipative forces*, going beyond the gyroscopic ones used in standard IDA-PBC. The contribution of our work is the proof that the addition of these two elements provides a non-trivial extension to the basic IDA-PBC methodology. It is also shown that several new controllers for mechanical systems designed invoking other (less systematic procedures) that do not satisfy the conditions of standard IDA-PBC, actually belong to this new class of SIDA-PBC.

1. Introduction

Stabilization of underactuated mechanical systems shaping their potential energy function, and preserving the systems structure, is a simple, robust and highly successful technique first introduced in [1]. To enlarge its realm of application it has been proposed to modify the kinetic energy of the system as well. This idea of total energy shaping was first introduced in [2] with the two main approaches being now: the method of controlled Lagrangians [3] and Interconnection and Damping Assignment Passivity-Based Control (IDA-PBC) [4], see also the closely related work [5]. In both cases stabilization (of a desired equilibrium) is achieved identifying the class of systems – Lagrangian for the first method and Hamiltonian for IDA-PBC – that can possibly be obtained via feedback. The conditions under which such a feedback law exists are identified by the so-called *matching equations*, which are a set of quasi-linear partial differential equations (PDEs), that are naturally split into kinetic energy (KE-PDE) and potential energy (PE-PDE).

Although a lot of research effort has been devoted to the solution of the matching equations – see [6,7] for a recent survey of the existing results – this task remains the main stumbling block for the application of these methods. The solution of the KE-PDE is simplified by the inclusion of *gyroscopic forces* in the target dynamics, which translates into the presence of a free skew-symmetric matrix in the matching equation that reduces the number of PDEs to be solved. Due to its Hamiltonian formulation, this term is intrinsic in IDA-PBC, and was added to the original controlled Lagrangian method of [3,8] – for the first time in [9] – and adopted later in [10], as recognized in its introduction. In [9] it is shown that the PDEs of the (extended) controlled Lagrangian method and IDA-PBC are the same, see also [10].

Recently, in [11] it has been proposed to consider a more general form of gyroscopic forces, relaxing the skew-symmetry condition. However, it is shown in [6] that the inclusion of these forces *does not reduce the number* of KE-PDEs. One of the objectives of this paper is to show that, even though the number of PDEs is not reduced, the inclusion of dissipative forces effectively extends the realm of application of IDA-PBC. A second modification to IDA-PBC proposed in the paper is to simultaneously carry out the energy shaping and damping injection steps—instead of doing them as

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separate steps. This modification has been previously reported in [12,13], where it is shown that the partition into two steps of the design procedure induces some loss of generality. In particular, it is shown in [12] that (two-step) IDA-PBC is not applicable for the induction motor, while SIDA-PBC does apply; and the result in [13] suggests that SIDA-PBC may be needed for stabilization of underactuated mechanical systems with damping. In our work, the idea of performing energy shaping and damping injection simultaneously is tailored to PBC design of mechanical systems that, together with the use of dissipative forces, provides a non-trivial extension of the basic IDA-PBC methodology.

In the paper we also show that several recent controller designs that do not fit in the standard IDA-PBC paradigm, actually belong to this new class of SIDA-PBC with dissipative forces. In this way, it is shown that these controllers, that were derived invoking less systematic procedures, are obtained following the well-established SIDA-PBC methodology.

The remaining of the paper is organized as follows. Section 2 briefly recalls the IDA-PBC methodology. Section 3 contains the main result, which is the definition of SIDA-PBC by using dissipative forces. Two recently reported controller design techniques are shown to belong to this class in Section 4. The paper is wrapped-up with concluding remarks in Section 5.

Notation. I_n is the $n \times n$ identity matrix and $0_{n\times s}$ is an $n \times s$ matrix of zeros, 0_n is an n-dimensional column vector of zeros. Given $a_i \in \mathbb{R}$, $i \in \bar{n} := \{1, \dots, n\}$, we denote with $\mathrm{col}(a_i)$ the n-dimensional column vector with elements a_i. For any matrix $A \in \mathbb{R}^{n\times n}$, $(A)_i \in \mathbb{R}^n$ denotes the ith column, $(A)^i$ the ith row and $(A)_{ij}$ the ij-th element. $e_i \in \mathbb{R}^n$, $i \in \bar{n}$, are the Euclidean basis vectors. For $x \in \mathbb{R}^n$, $S \in \mathbb{R}^{n\times n}$, $S = S^\top > 0$, we denote the Euclidean norm $|x|^2 := x^\top x$, and the weighted-norm $\|x\|_S^2 := x^\top S x$. Given a function $f : \mathbb{R}^n \to \mathbb{R}$ we define the differential operators

$$\nabla_x f := \left(\frac{\partial f}{\partial x}\right)^\top, \qquad \nabla_{x_i} f := \left(\frac{\partial f}{\partial x_i}\right)^\top,$$

where $x_i \in \mathbb{R}^p$ is an element of the vector x. For a mapping $g : \mathbb{R}^n \to \mathbb{R}^m$, its Jacobian matrix is defined as

$$\nabla g := \begin{bmatrix} (\nabla g_1)^\top \\ \vdots \\ (\nabla g_m)^\top \end{bmatrix},$$

where $g_i : \mathbb{R}^n \to \mathbb{R}$ is the ith element of g. When clear from the context the subindex in ∇ will be omitted. To simplify the expressions, the arguments of all mappings will be omitted, and will be explicitly written only the first time that the mapping is defined.

2. Standard IDA-PBC for mechanical systems

To make the paper self-contained a brief review of IDA-PBC is presented in this section. IDA-PBC was introduced in [4] to control underactuated mechanical systems described in port-Hamiltonian (pH) form by

$$\Sigma : \begin{bmatrix} \dot q \\ \dot p \end{bmatrix} = \begin{bmatrix} 0_{n\times n} & I_n \\ -I_n & 0_{n\times n} \end{bmatrix} \nabla H(q,p) + \begin{bmatrix} 0_{n\times m} \\ G(q) \end{bmatrix} u, \tag{1}$$

where $q, p \in \mathbb{R}^n$ are the generalized position and momenta, respectively, $u \in \mathbb{R}^m$ is the control, $G : \mathbb{R}^n \to \mathbb{R}^{n\times m}$ with $\mathrm{rank}(G) = m < n$, the function $H : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}$,

$$H(q,p) := \frac{1}{2} p^\top M^{-1}(q)\, p + V(q) \tag{2}$$

is the total energy with $M : \mathbb{R}^n \to \mathbb{R}^{n\times n}$, the positive definite inertia matrix and $V : \mathbb{R}^n \to \mathbb{R}$ the potential energy. The control objective is to generate a state-feedback control that assigns to the closed-loop the stable equilibrium $(q,p) = (q^\star, 0)$, $q^\star \in \mathbb{R}^n$. This is achieved in IDA-PBC via a two step procedure. The first step is called energy shaping and the second step is called damping injection. The following proposition summarizes these two steps.

Proposition 1 ([4]). *Consider the system (1). Assume that there exist the following*

- *$M_d : \mathbb{R}^n \to \mathbb{R}^{n\times n}$ is positive definite,*
- *$J_2 : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^{n\times n}$ fulfills the skew-symmetry condition*

$$J_2(q,p) = -J_2^\top(q,p). \tag{3}$$

- *$V_d : \mathbb{R}^n \to \mathbb{R}$ verifies*

$$q_\star = \arg\min V_d(q), \tag{4}$$

and the minimum is isolated, that satisfies the PDEs

$$G^\perp \left[\nabla_q H - M_d M^{-1} \nabla_q H_d + J_2 M_d^{-1} p\right] = 0, \tag{5}$$

where $G^\perp : \mathbb{R}^n \to \mathbb{R}^{s\times n}$, $s := n - m$ is a full rank left annihilator of G, i.e., $G^\perp G = 0_{s\times m}$ and $\mathrm{rank}(G^\perp) = s$. Then,

(i) *(Energy shaping) The system (1) in closed-loop with the control law $u = u_{\mathrm{ES}}(q,p)$, where $u_{\mathrm{ES}} : \mathbb{R}^{n\times n} \to \mathbb{R}^m$ is defined as*

$$u_{\mathrm{ES}} = (G^\top G)^{-1} G^\top \left[\nabla_q H - M_d M^{-1} \nabla_q H_d + J_2 M_d^{-1} p\right], \tag{6}$$

can be written as a pH system

$$\Sigma_d : \begin{bmatrix} \dot q \\ \dot p \end{bmatrix} = \begin{bmatrix} 0_{n\times n} & M^{-1}(q) M_d(q) \\ -M_d(q) M^{-1}(q) & J_2(q,p) \end{bmatrix} \times \nabla H_d(q,p) \tag{7}$$

with the new total energy function $H_d : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}$,

$$H_d(q,p) := \frac{1}{2} p^\top M_d^{-1}(q)\, p + V_d(q). \tag{8}$$

Therefore $(q^\star, 0)$ is a stable equilibrium point of (7) with Lyapunov function H_d.

(ii) *(Damping injection) Consider the mapping $u = u_{\mathrm{DI}}(q,p)$, where $u_{\mathrm{DI}} : \mathbb{R}^{n\times n} \to \mathbb{R}^m$ is defined as*

$$u_{\mathrm{DI}} = -K_p G^\top M_d^{-1} p, \tag{9}$$

with $K_p \in \mathbb{R}^{m\times m}$ positive definite. Then, the system (1) in closed-loop with the control $u = u_{\mathrm{ES}}(q,p) + u_{\mathrm{DI}}(q,p)$ has an asymptotic stable equilibrium $(q^\star, 0)$ provided that the output $G^\top M_d^{-1} p$ is detectable.

Proof. We show here a sketch of the proof in [4]. First, to prove (i), we equate the right-hand sides of (1) and (7) to obtain the so-called matching equations

$$\nabla_q H - G u = M_d M^{-1} \nabla_q H_d - J_2 M_d^{-1} p. \tag{10}$$

These equations are equivalent to the solution of the PDEs (5) and the (univocally defined) control (6). Since, by assumption, the functions H, M, H_d, M_d and J_2 satisfy (5), then the closed-loop can be written in the form (7). To prove stability, we take H_d as a Lyapunov candidate function and we compute its time derivative along the trajectories of (7), which takes the form

$$\dot H_d = p^\top M_d^{-1} J_2 M_d^{-1} p \equiv 0.$$

By adding the damping injection term in the controller as in (ii), we obtain that

$$\dot H_d = -\|G^\top M_d^{-1} p\|_{K_p}^2 \le 0,$$

which ensures asymptotic stability if the output $G^\top M_d^{-1} p$ is detectable [14]. □