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Pinning bipartite synchronization for coupled nonlinear systems with antagonistic interactions and switching topologies

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This paper studies the bipartite synchronization in a network of nonlinear systems with antagonistic interactions and switching topologies. In order to obtain some conditions such that the network achieves bipartite synchronization for any initial conditions, we design a pinning scheme to pin a part of agents. Under the assumptions that all signed graphs are structurally balanced and the nonlinear system satisfies a one-sided Lipschitz condition, we derive conditions under which the network reaches bipartite synchronization for any initial conditions and arbitrary switching signals. For a general switching signal (especially the periodic switching signal), some conditions related to switching signal are obtained. Finally, we present two numerical examples to illustrate the effectiveness of the obtained results.

1. Introduction

The study of synchronization (consensus) among a group of agents using only local actions has been an active area of research in systems and control during the last decade, thanks to many applications in physics, biology, society, and engineering. Although the synchronization problem in a network of systems over nonnegative graphs in which the interactions among agents are cooperative has been extensively studied (see [1–7] and references therein), antagonists (competition) is another inherent relationship among agents in social, biological and information fields, such as social networks [8,9], competing species and competitive cellular neurons [10–12], and personalized recommendation [13,14]. In this case, the interactions among agents can be presented as a signed graph. There have been some papers related to the synchronization in a network of systems over signed graphs [15–21].

When there coexist both collaborative and antagonistic (competitive) relationships in a network, Altafini first introduced the bipartite consensus concept (all agents converge to a value which is the same for all in modulus but not in sign) and proved that the network admits a bipartite consensus solution if and only if the signed graph is structurally balanced [15]. This result has been extended to general linear multi-agent systems [18,17] and dynamic output feedback control [19], where each agent is modeled by a linear time-invariant (LTI) system. Under a weak connectivity assumption that the signed network has a spanning tree, the Refs. [16,21] obtained some sufficient conditions for bipartite consensus of multi-integrator systems on directed signed networks. In addition, the authors of [20] studied the bipartite flock control problem in multi-integrator systems.

It is well known that almost all real systems are nonlinear. When the dynamics of agents are nonlinear, the synchronization problem in a network of systems over signed graphs becomes more complex. For example, suppose that the origin is an equilibrium of the network and it is a hyperbolic equilibrium. In order to make the network achieve bipartite synchronization, the initial conditions cannot lie in stable manifold of the origin. If there exists a homoclinic orbit which joins the origin to itself, then the initial conditions cannot lie in homoclinic orbit of the origin. In addition, if there exists a heteroclinic orbit which joins the other equilibrium to the origin, then the initial conditions cannot lie in heteroclinic orbit. Hence, if one wants to study the bipartite synchronization problem in a network of nonlinear systems over signed graphs, then one must consider various cases. In [22], we studied the modulus synchronization in a network of nonlinear systems with antagonistic interactions and switching topologies. Inspired by pinning control [23–26], some controllers may be designed and applied to force the network to achieve bipartite synchronization. A natural question to ask is the following: What kinds of pinning schemes may be designed to ensure the network bipartite synchronization? For the network of nonlinear systems with antagonistic interactions and switching topologies, we design

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a switching pinning control such that the controlled network achieves bipartite synchronization for any initial conditions. More precisely, we first obtain some conditions such that the controlled network achieves bipartite synchronization for any initial conditions and switching signals. Then, for a general switching signal, we obtain appropriate conditions related to switching signal to ensure bipartite synchronization for any initial conditions. Specially, when the switching signal is periodic, an easy checkable condition is obtained.

The paper is organized as follows. Section 2 presents problem statement. Section 3 presents some theorems and corollaries about the pinning bipartite synchronization in a network of nonlinear systems with antagonistic interactions and switching topologies. Section 4 shows some numerical examples to verify the obtained results. Section 5 summarizes our conclusions and describes future work. Appendix presents some definitions and facts about signed graphs.

The following notations are used throughout this article. The notation $|x|$ denotes the absolute value of real number x. The symbol I_N denotes the N-dimensional identity matrix, and the operator $\otimes$ denotes the Kronecker product. The notation $P > 0(<0)$ means P is real symmetric and positive definite (negative definite). For a real symmetric matrix A, $\lambda_{\max}(A)$ and $\lambda_{\min}(A)$ denote the maximum and minimum eigenvalues of A, respectively.

2. Problem statement

Consider a family of network of nonlinear dynamics with antagonistic interactions

$$\dot{x}_i(t) = f(x_i(t)) + c\sum_{j=1}^{N}\left|a_{ij}^k\right|\left(\operatorname{sgn}(a_{ij}^k)x_j(t) - x_i(t)\right), \quad k \in \mathcal{P}, \quad (1)$$

where $i = 1, 2, \ldots, N$, $x_i \in \mathbb{R}$ is the state of the ith agent, $f : \mathbb{R} \to \mathbb{R}$ is a smooth odd function, c is a positive constant, sgn is signature function, and $\mathcal{P} := \{1, 2, \ldots, p\}$ is an index set. For each $k \in \mathcal{P}$, $A^k = (a_{ij}^k) \in \mathbb{R}^{N\times N}$ is the adjacency matrix of signed graph $\mathcal{G}(A^k)$. A network of nonlinear dynamics with antagonistic interactions and switching topologies generated by the family of networks (1) and a switching signal $\sigma(t)$ is

$$\dot{x}_i(t) = f(x_i(t)) + c\sum_{j=1}^{N}\left|a_{ij}^{\sigma(t)}\right|\left(\operatorname{sgn}(a_{ij}^{\sigma(t)})x_j(t) - x_i(t)\right), \quad (2)$$

where $\sigma : [0, \infty) \to \mathcal{P}$ is a piecewise and right continuous constant function whose switching instants $\{t_i : i = 0, 1, \ldots\}$ satisfy $t_{i+1} - t_i \geq T_1 > 0$, $\forall i \geq 0$.

Throughout this paper, we have the following assumptions:

Assumption 1. Suppose that the function $f(x)$ in Eq. (1) satisfies the one-sided Lipschitz condition, i.e., $\forall x, y \in \mathbb{R}$, $(x - y)(f(x) - f(y)) \leq m(x - y)^2$, where m is a positive constant.

Assumption 2. Assume that all signed graphs $\mathcal{G}(A^k)$, $k = 1, \ldots, p$ are connected, and the vertices of graphs $\mathcal{G}(A^k)$, $k = 1, \ldots, p$ can be partitioned into two subsets V_1, V_2 such that $a_{ij}^k \geq 0$, $\forall i, j \in V_{\ell_1}, \ell_1 = 1, 2$ and $a_{ij}^k \leq 0$, $\{(i, j)| \ i \in V_{\ell_1}, j \in V_{\ell_2}, \ell_1, \ell_2 \in \{1, 2\}, \ell_1 \neq \ell_2\}$.

We consider a solution $s(t)$ of an isolated agent which satisfies

$$\dot{s}(t) = f(s(t)). \quad (3)$$

Here, $s(t)$ can be viewed as the state of the leader. We assume that the leader can pin either V_1 or V_2. Without loss of generality, suppose that $V_1 = \{1, 2, \ldots, l\}$, $V_2 = \{l + 1, \ldots, N\}$ and the pinning strategy is applied on V_1. Hence, the controlled coupled nonlinear systems can be presented as

$$\begin{cases} \dot{x}_i(t) = f(x_i(t)) + c\sum_{j=1}^{N}\left|a_{ij}^{\sigma(t)}\right|\left(\operatorname{sgn}(a_{ij}^{\sigma(t)})x_j(t) - x_i(t)\right) + u_i, \\ \quad i = 1, 2, \ldots, l, \\ \dot{x}_i(t) = f(x_i(t)) + c\sum_{j=1}^{N}\left|a_{ij}^{\sigma(t)}\right|\left(\operatorname{sgn}(a_{ij}^{\sigma(t)})x_j(t) - x_i(t)\right), \\ \quad i = l + 1, \ldots, N, \end{cases} \quad (4)$$

where

$$u_i = c\beta_i^{\sigma(t)}(x_i(t) - s(t)), \quad i = 1, 2, \ldots, l, \quad (5)$$

and β_i^k, $k = 1, 2, \ldots, p$ are needed to design.

The objective of this paper is to design appropriate controllers u_i such that the controlled coupled nonlinear systems (4) achieve pinning bipartite synchronization in the sense that

$$\begin{cases} \lim_{t\to\infty}(x_i(t) - s(t)) = 0, & \forall i \in V_1 \\ \lim_{t\to\infty}(x_i(t) + s(t)) = 0, & \forall i \in V_2 \end{cases} \quad (6)$$

for some nontrivial trajectory $s(t)$.

3. Main results

In this section, some results about pinning bipartite synchronization will be presented. The following theorem states that the coupled nonlinear systems (2) can reach pinning bipartite synchronization by an appropriate pinning strategy (5) under arbitrary switching signals.

Theorem 1. *Consider the coupled nonlinear systems (2), and there exists $\epsilon > 0$ such that $f(x) > 0$, $\forall x \in (0, \epsilon)$ and $f(x) < 0$, $\forall x \in (-\epsilon, 0)$. Suppose that Assumptions 1 and 2 hold. If $s(0) \neq 0$ and there exist β_i^k, $i = 1, 2, \ldots, l$ such that*

$$mI_N - c\Psi L^k\Psi + cD^k < 0, \quad (7)$$

where $k = 1, 2, \ldots, p$,

$$\Psi = diag\{\underbrace{1, 1, \ldots, 1}_{l}, \underbrace{-1, \ldots, -1}_{N-l}\},$$

$$D^k = diag\{\beta_1^k, \beta_2^k, \ldots, \beta_l^k, 0, \ldots, 0\},$$

then the controlled coupled nonlinear systems (4) achieve pinning bipartite synchronization under arbitrary switching signals $\sigma(t)$.

Proof. Since Assumption 2 holds, all signed graphs $\mathcal{G}(A^k)$, $k = 1, \ldots, p$ are connected and structurally balanced. One can know from Lemma 1 that there exists a common signature matrix $\Psi \in \ominus$ such that all matrices $\Psi A^k\Psi$, $k = 1, \ldots, p$ have nonnegative entries. Let the signature matrix $\Psi = \operatorname{diag}\{\psi_1, \psi_2, \ldots, \psi_N\}$. Then one can obtain from Lemma 2 that $\psi_i = 1$, $i = 1, \ldots, l$ and $\psi_i = -1$, $i = l + 1, \ldots, N$.

Let $x = [x_1 \cdots x_N]^{\mathrm{T}}$, $z = [z_1 \cdots z_N]^{\mathrm{T}}$. Consider the change of coordinates $z = \Psi x$, then the controlled coupled nonlinear systems (4) become

$$\begin{cases} \dot{z}_i(t) = f(z_i) + c\psi_i\sum_{j=1}^{N}\left|a_{ij}^{\sigma(t)}\right|\left(\operatorname{sgn}(a_{ij}^{\sigma(t)})\psi_j z_j - \psi_i z_i\right) + \psi_i u_i, \\ \quad i = 1, 2, \ldots, l, \\ \dot{z}_i(t) = f(z_i) + c\psi_i\sum_{j=1}^{N}\left|a_{ij}^{\sigma(t)}\right|\left(\operatorname{sgn}(a_{ij}^{\sigma(t)})\psi_j z_j - \psi_i z_i\right), \\ \quad i = l + 1, \ldots, N, \end{cases} \quad (8)$$