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Stabilization of retarded systems of neutral type by control Lyapunov–Krasovskii functionals☆

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ABSTRACT

This paper deals with the stabilization and the practical stabilization of nonlinear systems described by neutral functional differential equations in Hale's form, affine in the control input. Artstein's methodology and Sontag's universal formula are investigated for this class of systems, by means of invariantly differentiable control Lyapunov–Krasovskii functionals.

1. Introduction

Neutral functional differential equations in Hale's form (see [1,2]) describe many systems in electrical engineering (lossless transmission lines, partial element equivalent circuits, see [3,1,2,4] and references therein), mechanical engineering (hydraulic systems, see [4] and references therein), bioengineering (see [5] and references therein), and often provide an alternative description of systems described by hyperbolic partial differential equations (see [6] and references therein). Stabilization and control problems for classes of nonlinear systems described by neutral functional differential equations have been extensively treated (see, for instance, [7–12,5,13–16]). On the other hand, the stabilization approach based on the Artstein–Sontag Control Lyapunov Functions methodology (see [17,18]), to my knowledge, has been never investigated in the literature for systems described by neutral functional differential equations. As well known, the Control Lyapunov Functions (or Functionals) methodology has revealed to be a powerful and very general tool in nonlinear control of finite and also infinite dimensional systems including systems described by retarded functional differential equations (see [19–25] and references therein). It is easy to believe that this approach can play a key role in the stabilization theory for systems described by nonlinear neutral functional differential equations as well.

This paper aims to explore the Artstein–Sontag methodology for systems described by nonlinear neutral functional differential equations in Hale's form, affine in the control input, with an arbitrary number of discrete time-delays of arbitrary size. Partially distributed delay terms are also allowed. Two kinds of results are provided. First, Sontag's universal formula for neutral systems and the related small-control property are properly reformulated by means of new defined invariantly differentiable control Lyapunov–Krasovskii functionals (see [26,27]). It is shown that the resulting feedback control law is locally Lipschitz away from zero and continuous at zero, and yields global asymptotic stability of the trivial solution. This technique of Sontag's universal formula obtained by invariantly differentiable functionals, without any approximation, is not shown in [24], not even for systems described by retarded functional differential equations. The hypotheses needed for achieving a continuous global stabilizer by Sontag's formula and invariantly differentiable functionals are given, and it is shown by a counter-example that these hypotheses in general cannot be weakened for achieving this task (see Remark 10, where a comparison with other hypotheses introduced in [20–22,25], for systems described by retarded functional differential equations, is also provided). Sontag's universal formula and the small control property for systems described by neutral functional differential equations (which include as special case the class of systems described by retarded functional differential

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equations) are here provided in a form very similar to the one in the literature for systems described by ordinary differential equations. Second, a practically stabilizing controller for systems described by neutral functional differential equations is proposed by a modification of Sontag's universal stabilizer, as shown in [24] for nonlinear retarded systems. The feedback control law is locally Lipschitz, and in this case, the small-control property is not required. Instead, another condition is required to be verified (see point (iv) in Hypothesis 18). The price to pay is that only practical stability is guaranteed. On the other hand, the final target neighborhood of the origin can be arbitrarily small. An example is studied in details, showing the efficacy of the above modified Sontag's stabilizer.

In general, the extension of results from systems described by retarded functional differential equations to systems described by neutral functional differential equations has to be carefully managed, and devoted literature is necessary. For instance, in our case, it is necessary to introduce a new definition of invariantly differentiable functionals. As well, many of the results provided here exploit suitably the input-to-state stability property of the nonlinear difference operator, which is not present in systems described by retarded functional differential equations. Throughout the paper, for reader's convenience, differences between the two classes of systems and related methodologies are highlighted.

The paper is organized as follows. In Section 2, the definition of invariantly differentiable functionals (given in [26,27] for systems described by retarded functional differential equations) is given for systems described by neutral functional differential equations in Hale's form. In Section 3, standard Sontag's stabilizer, built up by the use of invariantly differentiable functionals, is investigated. In Section 4, an approximated Sontag's stabilizer is proposed, for the case that the small-control property is not satisfied, providing a new chance for stabilization, but of the practical type. In Section 5 an example is provided. In Section 6 conclusions are drawn.

A preliminary version of this paper has been published in [28].

Notations

R denotes the set of real numbers, $R^\star$ denotes the extended real line $[-\infty, +\infty]$, R^+ denotes the set of non negative reals $[0, +\infty)$. The symbol $|\cdot|$ stands for the Euclidean norm of a real vector, or the induced Euclidean norm of a matrix. For a positive integer n, for a positive real Δ (maximum involved time-delay), $\mathcal{C}$ and $\mathcal{Q}$ denote the space of the continuous functions mapping $[-\Delta, 0]$ into R^n and the space of the continuous functions mapping $[-\Delta, 0)$ into R^n, admitting finite left-hand limit at 0, respectively. The supremum norm of a function in $\mathcal{C}$, or in $\mathcal{Q}$, is indicated with the symbol $\|\cdot\|_\infty$. For $\phi \in \mathcal{C}$, ϕ_- is the function in $\mathcal{Q}$ defined as $\phi_-(\tau) = \phi(\tau)$, $\tau \in [-\Delta, 0)$. For a function $x : [-\Delta, c) \to R^n$, with $0 < c \le +\infty$, for any real $t \in [0, c)$: x_t is the function in $\mathcal{C}$ defined as $x_t(\tau) = x(t+\tau)$, $\tau \in [-\Delta, 0]$; x_{t^-} is the function in $\mathcal{Q}$ defined as $x_{t^-}(\tau) = x_t(\tau)$, $\tau \in [-\Delta, 0)$. For a positive real δ, $\phi \in \mathcal{C}$, $\mathcal{C}_\delta(\phi) = \{\psi \in \mathcal{C} : \|\psi - \phi\|_\infty \le \delta\}$. For a positive real δ, $x \in R^n$, $B_\delta(x) = \{y \in R^n : |y - x| \le \delta\}$. Let us here recall that a function $\gamma : R^+ \to R^+$ is: of class $\mathcal{P}$ if it is continuous, zero at zero, and positive at any positive real; of class $\mathcal{K}$ if it is of class $\mathcal{P}$ and strictly increasing; of class $\mathcal{K}_\infty$ if it is of class $\mathcal{K}$ and it is unbounded; of class $\mathcal{L}$ if it is continuous and it monotonically decreases to zero as its argument tends to $+\infty$. A function $\beta : R^+ \times R^+ \to R^+$ is of class $\mathcal{KL}$ if $\beta(\cdot, t)$ is of class $\mathcal{K}$ for each $t \ge 0$ and $\beta(s, \cdot)$ is of class $\mathcal{L}$ for each $s \ge 0$. A function $f : R^n \times \mathcal{Q} \to R^n$ is said to be independent of the second argument at 0, if there exists a positive real $\delta \in (0, \Delta)$ such that for any $x \in R^n$, for any $\phi_i \in \mathcal{Q}$, $i = 1, 2$, satisfying $\phi_1(\tau) = \phi_2(\tau)$, $\tau \in [-\Delta, -\delta]$, the equality holds $f(x, \phi_1) = f(x, \phi_2)$ (see Definition 5.1, p. 281, in [1,29]). The symbols $\cup$ and $\circ$ denote union of sets and composition of functions, respectively. In next sections, ODE stands for ordinary differential equation, RFDE stands for retarded functional differential equation, F$\mathcal{D}$E stands for functional difference equation, NFDE stands for neutral functional differential equation, GAS stands for global asymptotic stability or globally asymptotically stable, ISS stands for input-to-state stability or input-to-state stable, CLKF stands for control Lyapunov–Krasovskii functional. We recall that: a system is said to be 0-GAS if the origin is an equilibrium point and the null solution is GAS (see [30], here the origin is considered as zero-invariant set); a map $f : \mathcal{C} \to R^n$ is said to be completely continuous if it is continuous and it takes closed bounded sets in $\mathcal{C}$ into bounded sets of R^n (see [1, Theorem 3.2, pp. 46]).

2. Invariantly differentiable functionals for NFDEs

Let us consider a system described by the following NFDE in Hale's form (see [1,29])

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathcal{D}x_t = f(x(t), x_{t^-}) + g(x(t), x_{t^-})u(t), \quad t \ge 0,$$
$$x(\tau) = x_0(\tau), \quad \tau \in [-\Delta, 0],\ x_0 \in \mathcal{C}, \tag{1}$$

where: $x(t) \in R^n$, n is a positive integer; $\Delta > 0$ is the maximum involved time-delay; the maps $f : R^n \times \mathcal{Q} \to R^n$ and $g : R^n \times \mathcal{Q} \to R^{n\times m}$ are Lipschitz on bounded subsets of $R^n \times \mathcal{Q}$ and independent of the second argument at 0; m is a positive integer; $u(t) \in R^m$ is the input signal, Lebesgue measurable and locally essentially bounded; $\mathcal{D} : \mathcal{C} \to R$ is a map defined as follows, for $\phi \in \mathcal{C}$,

$$\mathcal{D}\phi = \phi(0) - q(\phi_-), \tag{2}$$

where the map $q : \mathcal{Q} \to R^n$ is Lipschitz on bounded subsets of $\mathcal{Q}$, independent of the argument at 0. We assume $f(0, 0) = 0$ and the existence of a function L of class $\mathcal{K}_\infty$ such that

$$|q(\phi)| \le L(\|\phi\|_\infty), \quad \forall\, \phi \in \mathcal{Q}. \tag{3}$$

Moreover, we introduce the following assumption (not involved for systems described by RFDEs).

Assumption 1. There exist functions $\tilde{\beta}$ of class $\mathcal{KL}$ and $\tilde{\gamma}$ of class $\mathcal{K}$ such that, for the solution of the system described by the F$\mathcal{D}$E

$$\mathcal{D}\xi_t = w(t), \quad t \ge 0,$$
$$\xi(\tau) = \xi_0(\tau), \quad \tau \in [-\Delta, 0],\ \xi_0 \in \mathcal{C}, \tag{4}$$

with $\xi(t) \in R^n$, $w(t)$ a continuous input signal, the inequality holds

$$\|\xi_t\|_\infty \le \tilde{\beta}(\|\xi_0\|_\infty, t) + \tilde{\gamma}\left(\sup_{\tau\in[0,t]} |w(\tau)|\right) \tag{5}$$

(that is, the system described by the F$\mathcal{D}$E (4) is ISS, see [31–33]).

Remark 2. If, in (2), $q(\phi_-) \equiv 0$, $\phi \in \mathcal{C}$ (i.e., $\mathcal{D}\phi = \phi(0)$), then (1) reduces to a RFDE.

The definition of invariant differentiable functionals is given in [26,27] for systems described by RFDEs (see Definitions 2.2.1, 2.5.2 in Chapter 2 in [27]). The formalism used in [27] is here slightly modified for the purpose of formalism uniformity over the paper. We make suitable modifications to the Definitions in [27], in order to cope with systems described by NFDEs. In forthcoming Remark 4 a discussion is provided about differences between the definition here given for NFDEs and the one given in [27] for RFDEs. For any given $y \in R^n$, $\phi \in \mathcal{Q}$ and for any given continuous function $\mathcal{Y} : [-2\Delta, -\Delta] \cup [0, \Delta] \to R^n$ with $\mathcal{Y}(0) = y$, $\mathcal{Y}(-\Delta) = \phi(-\Delta)$, let $\psi_h^{(y,\phi,\mathcal{Y})} \in \mathcal{Q}$, $h \in (-\Delta, \Delta)$, be defined as:

$$\psi_0^{(y,\phi,\mathcal{Y})} = \phi;$$

$$\text{for } h > 0, \quad \psi_h^{(y,\phi,\mathcal{Y})}(s) = \begin{cases} \phi(s+h), & s \in [-\Delta, -h), \\ \mathcal{Y}(s+h), & s \in [-h, 0) \end{cases}$$

$$\text{for } h < 0, \quad \psi_h^{(y,\phi,\mathcal{Y})}(s) = \begin{cases} \phi(s+h), & s \in [-\Delta-h, 0), \\ \mathcal{Y}(s+h), & s \in [-\Delta, -\Delta-h). \end{cases} \tag{6}$$