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Multirate forward-model disturbance observer for feedback regulation beyond Nyquist frequency

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ABSTRACT

A fundamental challenge in digital control arises when the controlled plant is subjected to a fast disturbance dynamics but is only equipped with a relatively slow sensor. Such intrinsic difficulties are, however, commonly encountered in many novel applications such as laser- and electron-beam-based additive manufacturing, human–machine interaction, etc. This paper provides a discrete-time regulation scheme for exact sampled-data rejection of disturbances beyond Nyquist frequency. By introducing a model-based multirate predictor and a forward-model disturbance observer, we show that the inter-sample disturbances can be fully attenuated despite the limitations in sampling and sensing. The proposed control scheme offers several advantages in stability assurance and lucid design intuitions. Verification of the algorithm is conducted on a motion control platform that shares the general characteristics in several advanced manufacturing systems.

1. Introduction

A fundamental challenge arises in feedback control if the sampling of the output is not fast enough to capture the major frequency components of the disturbances—or more specifically, when significant disturbances occur beyond Nyquist frequency. Such a scenario is, however, becoming increasingly important in modern control systems, due to, on the one hand the continuous pursuit of higher performance and robustness using slow or limited sensing mechanisms (e.g., vision servo, chemical process, human–machine interaction, etc.); and on the other hand, many hardware constraints in novel applications such as in-situ sensing in additive manufacturing [1] and intrinsic numerical limitations [2] in ultra fast sampling. In these applications and the like, significant disturbances beyond Nyquist frequency are unattended under conventional servo design. Such large intersample/hidden disturbances are extremely dangerous, as they cause unobserved performance loss in the actual system, increase system fatigue, and can even lead to hardware failures. As a particular example, for advanced manufacturing such as the laser-based additive manufacturing process [1], there are significant challenges and opportunities for high-speed high-precision sensing and metrology. For instance, [3,4] used an infrared camera for the sensing and control of the molten-pool profile in laser cladding. The dynamics of laser melting is very fast. Although using a high-speed camera at 800 frames per second, the control of the closed-loop is limited at 30 Hz, as it takes time for the raw image data to be processed and for the signature characteristics in the molten pool to be estimated. Overcoming the limitations from slow/limited sampling is thus key for unlocking the full potentials of performance and robustness in the next-generation additive manufacturing.

From the viewpoint of information recovery, the pioneering work of Shannon [5] – now covered in standard text books such as [6] – has shown that in order for the original analog information to be fully recovered from its samples, (i) the analog signal must be perfectly band-limited below Nyquist frequency, and (ii) an ideal lowpass filter (acausal and not interpretable using a transfer function) is available in the reconstruction process. Recent advances using sampled-data H_∞ theory have investigated broadband reconstruction to approximate the ideal digital-to-analog converter (DAC) [7]. Although perfect recovery is still theoretically not feasible, the H_∞ approach provides an optimal tool focusing on the actual analog performance. For other approximate reconstructions, one is referred to the survey in [8].

From the viewpoint of control design, multirate control and advanced DAC have been the main tools for tracking and regulation beyond Nyquist frequency. Let the plant output be sampled at T_s s. Ref. [9] introduced a multirate feedforward control for exact tracking of the reference at the T_s-sampled instances. Optimization-based multirate iterative learning control with consideration of the inter-sample behavior was discussed in [10]. In feedback

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algorithms, partial compensation and control of the inter-sample system behaviors have been investigated. [11] used generalized predictive control under quadratic cost functions on hard disk drive systems; asynchronized sampling and generalized holders were applied to improve the inter-sample behavior in repetitive control [12]; and [13] discussed beyond-Nyquist servo control via peak filters that are discretized at a sampling time smaller than T_s. Along the direction of changing the DAC, generalized sample hold control has shown to be promising in sampled-data performance. However, the technique also has fundamental limitations in closed-loop robustness and sensitivity, which raised intrinsic difficulties for implementation in practice [14]. The introduction of the lifting technique [15] has been a major enabler for analysis of the inter-sample behavior. Under this scope, the challenge of inter-sample ripples has been shown to occur due to non-uniform gains of the discrete lifted system and inverse lifting [16,17]. A synthesis based on lifting and internal model principle is provided in [18], where the internal model is embedded in a H_∞ synthesis to certify the continuous-time performance.

The goal of this paper is to introduce a shifted paradigm that goes beyond the acausal, non-ideal information recovery and partial feedback disturbance compensation. We provide a mixed-rate feedback solution for the missing piece of exact disturbance rejection at frequencies beyond the Nyquist limitation. This is achieved by the introduction of a multirate forward-model disturbance observer (MR-FMDOB) that enables full rejection of structured disturbances at both the sampling and any uniformly spaced inter-sample instances. Built on top of a baseline controller at the regular sampling time of T_s, such an exact compensation scheme constructs an internal feedback loop implemented at a higher sampling rate, where the inter-sample signals are constructed with model-based prediction using the slow T_s-sampled data. Integrating features of all-stabilizing parameterization [19,20], the internal add-on loop guarantees the overall closed-loop stability and offers the convenience of being decoupled from the design of the baseline sub-Nyquist controller. Similar to [18], an internal model of the disturbance is implicitly integrated in the inner loop. The major differences here are the decoupled design of the fast-rate MR-FMDOB from the slow-rate baseline feedback control, and the all-stabilizing based observer structure that preserves features of the baseline servo. These properties enable additional design freedom such as adaptive control in the baseline or the multirate compensator, and are also useful for applications where the existing baseline servo has special features that are required to be preserved.

Notations: $\mathbb{R}$ denotes the set of real numbers. $\mathbb{Z}$ and $\mathbb{Z}^+$ denote, respectively, the sets of integers and positive integers. We use $x[n]$ and $x_c(t)$ to represent a discrete sequence and a continuous-time signal, respectively. $\mathscr{H}$ denotes a zero order hold (ZOH) whose transfer function is $H(s) = (1 - e^{-sT_s})/s$, if the sampling time is T_s.

2. Problem formulation

Consider the system in Fig. 1, where the solid and dashed lines represent, respectively, continuous- and discrete-time signal flows. The main elements here include the continuous-time plant $P_c(s)$, the sampler $\mathscr{S}$ at a sampling time of T_s s, the discrete-time controller $C(z)$, and the signal holder $\mathscr{H}$. Given the common hardware complication and the theoretical limitation [14] of generalized hold functions, we assume that the DAC implements a ZOH throughout the paper.

Throughout the paper, we assume that the plant has no hidden modes and the closed loop satisfies the nonpathological sampling condition [21]:

Assumption 1. $P_c(s) = P_0(s)e^{-s\tau}$ where $\tau \geq 0$; $P_0(s)$ and $C(z)$ are both linear time invariant (LTI), irreducible, proper, and rational.

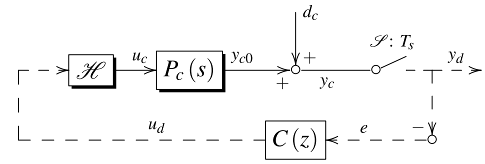

Fig. 1. Block diagram of a sampled-data control system.

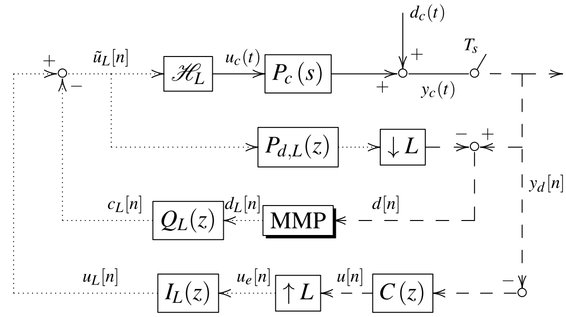

Fig. 2. The proposed multirate disturbance rejection scheme.

Assumption 2 (*Nonpathological Sampling*)**.** Let the minimal state equation of the plant be $\dot{x} = Ax + Bu_c$. Take any two eigenvalues λ_i and λ_j of A, with $\mathbf{Re}(\lambda_i) = \mathbf{Re}(\lambda_j)$. It is assumed that $\mathbf{Im}(\lambda_i) - \mathbf{Im}(\lambda_j) \neq 2\pi p/T_s$ for any nonzero integer p.

Under Assumption 2, sampling preserves controllability and observability, and the closed-loop sampled-data system is stable if and only if the discrete-time closed loop consisting of $C(z)$ and the ZOH equivalent of $P_c(s)$ is stable (see, e.g. [22–24]).

The focused problem is as follows. Suppose an additional fictitious faster sensor, at a sampling time of $T_s' = T_s/L$ ($L \in \mathbb{Z}^+$), is available between y_c and $\mathscr{S}$ in Fig. 1. We provide a control algorithm such that $y_c(nT_s')$ ($n \in \mathbb{Z}$) asymptotically converges to zero in the presence of a disturbance with significant spectrum beyond the Nyquist frequency $[1/(2T_s)$ Hz].

3. Main results

Fig. 2 shows the proposed servo scheme for disturbance rejection beyond Nyquist frequency. Two groups of discrete signals are present, with their different sampling rates indicated by the dashed (slower) and dotted (faster) signal flows.

In the block diagram, the upsampler (between $u[n]$ and $u_e[n]$) and the interpolator $I_L(z)$ generate a fast signal $u_L[n]$ at a sampling time of T_s/L. Under a ZOH interpolator, $u_L[n] = u[n/L]$ when $n = 0, \pm L, \pm 2L, \ldots$ and $u_L[n] = u[k]$ when $kL < n < (k+1)L$, respectively. The upsampled signal then passes through $\mathscr{H}_L$, a T_s/L-based ZOH, with a transfer function $H_L(s) = (1 - e^{-sT_s/L})/s$.

The beyond-Nyquist disturbance rejection consists of two fast-sampling transfer functions $Q_L(z)$ and $P_{d,L}(z)$, a downsampling operator, and a multirate model-based predictor (MMP) between the downsampler and $Q_L(z)$. In the subsequent derivations, we show that although $y_d[n]$ only contains information sampled at T_s, the inter-sample information in $d_c(t)$ can be fully reconstructed with MMP in Fig. 2, if $d_c(t)$ satisfies a disturbance model; and in that case, $c_L[n]$ – the output of $Q_L(z)$ – can fully remove the effect of the beyond-Nyquist sampled disturbance at a fast sampling period of T_s/L.

3.1. Multirate forward-model disturbance observer

If the sampling time in Fig. 2 were T_s/L, the focused signal is the sampled output, and the downsampler and the MMP block were