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# On observer-based fault detection for nonlinear systems

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ABSTRACT

This paper addresses analysis and integrated design of observer-based fault detection (FD) for nonlinear systems. To gain a deeper insight into the observer-based FD framework, definitions and existence conditions for nonlinear observer-based FD systems are studied first. Then, a scheme for an integrated design of observer-based FD systems for affine nonlinear systems is proposed. Our work is considerably inspired by the study on input–output stability and stabilization of nonlinear systems. Examples are given at the end of the paper to illustrate the theoretical results.



## 1. Introduction

Associated with the increasing industrial demands on safety and reliability, observer-based fault detection and isolation (FDI) has attracted intensive attention in the past two decades. One of the most challenging topics in the FDI research and application areas is nonlinear observer-based FDI. A review of the literatures in the last two decades shows that the application of nonlinear observer theory built the main stream in the nonlinear observer-based FDI study in the 1990s [1]. In recent years, much attention has been paid to the application of those techniques to addressing nonlinear FDI issues. For instance, Fuzzy technique based FDI [2,3], adaptive fault diagnosis [4,5], LPV-based FDI [6,7] or sliding mode observer-based fault detection [8,9] as well as their successful applications [10,11] have been reported.

In this paper, we deal with analysis and design issues of nonlinear observer-based fault detection (FD) systems. Roughly speaking, an observer-based FD system consists of an observer-based residual generator, a residual evaluator and a decision maker with a threshold. Reviewing the publications on nonlinear observer-based FD studies reveals that the major research focus in this area is on the design of observer-based residual generators [1,12]. Serving as a major methodology, nonlinear observer theory is widely applied for the investigations in this thematic field. While the first studies have been mainly devoted to the application of feedback-based linearization and differential algebra techniques to residual generator design [13–15], and the geometric approach to nonlinear FDI [16], the recent research efforts address systems with a special class of nonlinearities, typically Lipschitz nonlinearity [17,18], sector bounded nonlinearity [19] or special types of control systems like nonlinear networked control system [20]. Differently, [21–23] have investigated residual evaluation, threshold setting and residual generator optimization issues for nonlinear observer-based FD systems. It can be observed that (i) only few of these studies have dealt with residual generator and evaluation as well as decision making in an integrated way and (ii) most of efforts have been made on the FD system design but only few on analysis issues.

Motivated by these observations, in the first part of this paper we study an essential analysis issue, the existence conditions of nonlinear observer-based FD systems. With the aid of input–output stability theory, the concepts output re-constructability and weak output re-constructability are introduced and, in this context, existence conditions for different types of nonlinear observer-based FD systems are investigated. The objective of this work is to gain a deeper insight into the fundamental properties of nonlinear observer-based FD systems, which may be useful for the development of nonlinear FD systems using some well established technologies. In the second part of our work, we propose a scheme for an integrated design of observer-based FD systems for a class of nonlinear systems, the so-called affine systems. For the purpose of attaining an efficient FD system, a dynamic threshold is first proposed by taking into account of the influence of the input variables on the residual signal with the help of $\mathcal{L}_2$ stability

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theory [24]. Moreover, different cases for the application of the observer-based FD system are discussed.

The paper is organized as follows. In Section 2, needed preliminaries are introduced and the problems to be addressed are formulated. Section 3 is devoted to the study on definitions and existence conditions of observer-based FD systems. The integrated design of observer-based FD systems for affine nonlinear systems is proposed in Section 4, and different cases are addressed. In Section 5, examples are provided to illustrate the theoretical results.

**Notations**: Standard notation is adopted in this paper. In addition, following definitions known in nonlinear stability theory are used. $\mathcal{R}_+ = [0, \infty)$. A function $\gamma : \mathcal{R}_+ \to \mathcal{R}_+$ is said to belong to class $\mathcal{K}$ if it is continuous, strictly increasing, and satisfies $\gamma(0) = 0$. A function $\beta : \mathcal{R}_+ \to \mathcal{R}_+$ is said to belong to class $\mathcal{L}$ if it is continuous, strictly decreasing, and satisfies $\lim_{s\to\infty} \beta(s) = 0$. A function $\phi(s, t) : \mathcal{R}_+ \times \mathcal{R}_+ \to \mathcal{R}_+$ is said to belong to class $\mathcal{KL}$ if for each fixed $t$ the function is of class $\mathcal{K}$ and for each fixed $s$ it is of class $\mathcal{L}$. $\|\cdot\|$ stands for the Euclidean norm of a vector in some Euclidean space; $\mathcal{B}_r := \{x \in \mathcal{R}^n : \|x\| \le r \text{ for some } r > 0\}$. $\mathcal{L}_2(0, \infty)$ is the space of functions $u : \mathcal{R}_+ \to \mathcal{R}^p$ which are measurable and satisfy $\int_0^\infty \|u(t)\|^2 dt < \infty$. $\mathcal{L}_{2,[0,\tau]}$-norm of $u(t)$ is defined and denoted by $\|u_\tau\|_2 = \left(\int_0^\tau \|u(t)\|^2 dt\right)^{1/2}$, and $\|u\|_\infty = \text{ess sup}\{\|u(t)\|, t \ge 0\}$. A function $f : \mathcal{R}^n \to \mathcal{R}$ is positive definite if $f(x) > 0$ for all $x > 0$, and $f(0) = 0$. By $V_{x,\hat{x}}(x, \hat{x})$ we denote

$$V_{x,\hat{x}}(x,\hat{x}) = \begin{bmatrix} V_x(x,\hat{x}) & V_{\hat{x}}(x,\hat{x}) \end{bmatrix} = \begin{bmatrix} \frac{\partial V(x,\hat{x})}{\partial x} & \frac{\partial V(x,\hat{x})}{\partial \hat{x}} \end{bmatrix}.$$

## 2. Preliminaries and problem formulation

Consider nonlinear systems described by

$$\Sigma : \dot{x} = f(x, u), \qquad y = h(x, u) \tag{1}$$

where $x \in \mathcal{R}^n, u \in \mathcal{R}^p, y \in \mathcal{R}^m$ denote the state, output and input vectors, respectively. $f(x, u)$ and $h(x, u)$ are continuously differentiable nonlinear functions with appropriate dimensions. The affine form of $\Sigma$,

$$\Sigma : \dot{x} = a(x) + B(x)u, \qquad y = c(x) + D(x)u \tag{2}$$

with $a(x), B(x), c(x)$ and $D(x)$ being continuously differentiable and of appropriate dimensions, is a class of nonlinear systems which are widely adopted in nonlinear system research. This class of nonlinear systems can be considered as a natural extension of linear time-invariant (LTI) systems

$$\dot{x} = Ax + Bu, \qquad y = Cx + Du \tag{3}$$

where $A \in \mathcal{R}^{n\times n}, B \in \mathcal{R}^{n\times p}, C \in \mathcal{R}^{m\times n}, D \in \mathcal{R}^{m\times p}$. In the FD study on LTI systems, faults are often modelled by extending (3) to

$$\dot{x} = Ax + Bu + E_w w, \qquad y = Cx + Du + F_w w$$

where $w \in \mathcal{R}^{k_w}$ represents the fault vector and $E_w, F_w$ are known matrices of appropriate dimensions. Analog to it, an extension of (1) to

$$\Sigma : \dot{x} = f(x, u, w), \qquad y = h(x, u, w) \tag{4}$$

is adopted for modelling nonlinear faulty systems.

A standard observer-based FD system consists of an observer-based residual generator, a residual evaluator and a decision maker with a threshold [25]. For LTI systems, the most widely applied type of residual generators is a full-order observer described by

$$\dot{\hat{x}} = A\hat{x} + Bu + Lr, \quad r = y - \hat{y}, \ \hat{y} = C\hat{x} + Du. \tag{5}$$

(5) is also called fault detection filter (FDF), where $L$ is the observer gain to be designed such that the FDF is stable and residual signal $r(t)$ satisfies

$$\forall u(t), x(0), \quad \lim_{t\to\infty} r(t) = 0. \tag{6}$$

A norm of $r(t)$, typically $\mathcal{L}_2$- or $\mathcal{L}_\infty$-norm, is adopted as residual evaluation function, which is defined by

$$J = \|r(t)\| \quad \text{or} \quad J = \|r(t)\|_\infty. \tag{7}$$

Let $J_{th} = \sup_{x_0, w=0} J$ be the threshold, which is interpreted as the maximum influence of $x(0) = x_0$ on the fault-free ($w(t) = 0$) residual vector $r(t)$. A simple form of detection logic is

$$\text{detection logic: } \begin{cases} J > J_{th} \Longrightarrow \text{faulty} \\ J \le J_{th} \Longrightarrow \text{fault-free.} \end{cases} \tag{8}$$

In this paper, two essential nonlinear FD issues will be addressed. The first one deals with definitions and the existence conditions of observer-based FD systems for nonlinear systems (1). We will apply the well-established input–output stability theory and the concept of weak detectability for nonlinear systems [26] to our study. It is worth to emphasize that our work is devoted to the overall observer-based FD system with a residual generator, an evaluator and a decision maker. The second issue is the design of nonlinear observer-based FD systems for affine systems (2). In this work, the $\mathcal{L}_2$-gain technique will serve as the investigation tool [24]. We will propose a design scheme and apply it to different design cases.

## 3. On observer-based FD systems

In this section, we introduce some basic definitions needed for constructing a nonlinear observer-based FD system and study the existence conditions.

### 3.1. On the construction of observer-based FD systems

For our purpose, we first introduce the configuration of the observer-based FD systems to be addressed in this paper, which consists of an observer-based residual generator, an evaluation function and a decision maker.

**Definition 1.** Given the nonlinear system (1), a system of the form

$$\dot{\hat{x}} = \phi(\hat{x}, u, y) \tag{9}$$

$$r = \varphi(\hat{x}, u, y) \tag{10}$$

is called observer-based residual generator if it delivers a residual vector $r$ satisfying that (i) for $\hat{x}(0) = x(0), \forall u, r(t) \equiv 0$ (ii) for some $w \ne 0$ in the faulty system (4), $r(t) \ne 0$.

In order to avoid loss of information about the faults, the residual vector has generally the same dimension like the output vector. For the sake of simplicity, also considering the conditions (i) and (ii), we suppose that

$$r = \varphi(\hat{x}, u, y) = y - \hat{y}, \quad \hat{y} = h(\hat{x}, u). \tag{11}$$

For the residual evaluation purpose, two norm-based evaluation functions are considered in this paper: (i) the Euclidean norm-based instant evaluation

$$J_E = \alpha_1(\|r\|) \tag{12}$$

(ii) integral evaluation with an evaluation window $[0, \tau]$

$$J_2 = \int_0^\tau \alpha_2(\|r\|)\, dt \tag{13}$$

where $\alpha_1(\|r\|), \alpha_2(\|r\|)$ are some $\mathcal{K}$-functions. Moreover, the detection logic (8) will be used for decision making. To this end, threshold setting is a major part of constructing an observer-based FD system.