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Almost sure stability and stabilization of discrete-time stochastic systems

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ABSTRACT

As is well known, noise may play a stabilizing or destabilizing role in continuous-time systems. But, for analysis and design of discrete-time systems, noise is treated as disturbance in the literature. This paper studies almost sure stability of general n-dimensional nonlinear time-varying discrete-time stochastic systems and presents a criterion based on a numerical result derived from Higham (2001), which exploits the stabilizing role of noise in discrete-time systems. As an application of the established results, this paper proposes a novel controller design method for almost sure stabilization of linear discrete-time stochastic systems. The effectiveness of the proposed design method is verified with an example (an aircraft model subject to state-dependent noise), to which the existing results do not apply.

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1. Introduction

Over the past decades, stochastic systems described with stochastic differential/difference equations have been intensively studied since stochastic modeling has come to play an important role in science and engineering (see [1–10] and the references therein). In the literature, there are several different concepts of stability for stochastic systems such as stability in probability, pth moment stability and almost sure stability [1,7], while a major part of the works has been dedicated explicitly or implicitly to mean-square stability (see, e.g., [1,11,4–7,12,13] and the references therein). As far as mean-square stability is concerned, noise always plays a destabilizing role and hence is treated as disturbance since there is no way an unstable system can be stabilized by noise in the mean-square sense [8].

However, in the almost sure sense, noise can not only be used to destabilize a given stable system but also be used to stabilize a given unstable system or to make a system even more stable. The literature on stabilization and destabilization by noise is extensive (see [1,14–18,1,19,15] and the references therein). Among the key results, Mao [15] proposed a theory on stabilization and destabilization by Brownian motion for differential equations with the global Lipschitz condition; afterwards these results were significantly improved by Appleby et al. [18], which includes a much more general class of nonlinear dynamical systems; most recently, Huang [19] further developed the theory and revealed the more fundamental principle for stochastic stabilization and destabilization of nonlinear differential equations. These results on almost sure stability of stochastic differential equations have been applied to study feedback stabilization problems of stochastic systems, see, e.g., [8], where the results on mean-square stability may not be applicable.

It is noticed that almost sure stability and stabilization of stochastic differential equations, or say, continuous-time stochastic dynamical systems have received much attention while relatively few works (see, e.g., [3,9]) are concerned with those problems of stochastic difference equations, or say, discrete-time stochastic dynamical systems. As is known, whenever a computer is used in measurement, computation, signal processing or control applications, the data, signals and systems involved are naturally described with discrete-time processes, see, e.g., [20,21]. Therefore, theory of discrete-time dynamic signals and systems is useful in design and analysis of control systems, signal filters, and state estimators from time-series of process data as well as scientific computations. As a matter of fact, discrete-time stochastic systems and those discretized from continuous-time stochastic systems have

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been intensively studied over the past few decades, see [9,6,2,22,23,3,24,25,11,12] and the references therein. Most of these results study stochastic systems in mean-square sense since it is often relatively easier to analyze some properties such as stability and asymptotic behavior in this way [9]. But, as is well known, a mean-square unstable system could be almost surely asymptotically stable (or, simply, almost surely stable), which is concerned with all paths on the sample space and is more desired in many practical cases. It is also observed that some existing works focus on scalar systems (e.g., [9]) and few work investigates the almost sure stability of general n-dimensional time-varying discrete-time stochastic systems. This paper studies the almost sure stability of n-dimensional time-varying discrete-time stochastic systems (1) below. Based on a numerical result that is the development of a stability result for scalar linear time-invariant systems in [3], this paper establishes a criterion on almost sure stability of (1) and applies it to state-feedback controller design for almost sure stabilization of linear systems (48). The effectiveness of our proposed results are verified with application examples including an aircraft model subject to state-dependent noise, where the existing results do not work.

The rest of this paper is organized as follows: notation and preliminaries are given in Section 2; in Section 3, an important criterion for almost sure stability of general n-dimensional discrete-time (time-varying) stochastic systems (1) is established based on a numerical result; Section 4 applies this established result to linear discrete-time systems and proposes a novel design method of state-feedback controller for almost sure stabilization of linear discrete-time stochastic systems; concluding remarks on our proposed method are given in Section 5.

2. Preliminaries

Our problem will be embedded in an underlying complete probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_k\}_{k\geq 0}, \mathbb{P})$ with a natural filtration $\mathcal{F}_{k_1} \subset \mathcal{F}_{k_2}$ for $k_2 > k_1$, where Ω is the sample space, $\mathcal{F}$ is the σ-algebra that defines events E in Ω which are measurable, i.e., for which the probability $\mathbb{P}(E)$ is defined. Let $\mathbb{E}[\cdot]$ be the expectation operator with respect to the probability measure. Let $(\mathcal{F}_k, \mathcal{F}_k^+)$, $k \geq 0$, be a pair of families of σ-algebras such that (i) $\mathcal{F}_k \subset \mathcal{F}$ is monotone increasing, (ii) $\mathcal{F}_k^+ \subset \mathcal{F}$ is monotone decreasing, and (iii) $\mathcal{F}_k$ and $\mathcal{F}_k^+$ are independent for all $k \geq 0$. Throughout this paper, unless otherwise specified, the following notation shall be employed. Denote by $\mathbb{Z}$, $\mathbb{Z}_+$, $\mathbb{R}$ and $\mathbb{R}_+$ the sets of all integers, nonnegative integers, real and nonnegative real numbers, respectively. For real numbers x and y, denote by $x \vee y$ (resp. $x \wedge y$) the maximum (resp. minimum) of x and y while $\pm x \geq y$ (resp. $x \geq \pm y$) means that $x \geq y$ or $-x \geq y$ (resp. $x \geq y$ or $x \geq -y$). A function $f : \mathbb{R}^n \times \mathbb{Z}_+ \to \mathbb{R}^n$ is said to be of class $C_{\mathbb{Z}_+}(\mathbb{R}^n; Y)$ if, for each fixed $k \in \mathbb{Z}_+$, the function $f(\cdot, k)$ is a continuous mapping from $\mathbb{R}^n$ to $\mathbb{R}^n$. If A is a vector or matrix, its transpose is denoted by A^T. If P is a square matrix, $P > 0$ ($P < 0$) means that P is a symmetric positive (negative) definite matrix of appropriate dimensions while $P \geq 0$ (resp. $P \leq 0$) is a symmetric positive (resp. negative) semidefinite matrix. I stands for the identity matrix of appropriate dimensions. Denote by $\widetilde{\lambda}_M(\cdot)$ (resp. $\widetilde{\lambda}_m(\cdot)$) the maximum (resp. minimum) eigenvalue of a matrix. Let $|\cdot|$ denote the Euclidean norm of a vector or its induced norm of a matrix. Unless explicitly stated, matrices are assumed to have real entries and compatible dimensions.

Let us consider a discrete-time stochastic system

$$x_{k+1} = f(x_k, k) + g(x_k, k)w_{k+1}, \quad k \in \mathbb{Z}_+ \tag{1}$$

with initial condition $x_0 \in \mathbb{R}^n \setminus \{0\}$, where functions $f(\cdot,\cdot)$ and $g(\cdot,\cdot)$ are of class $C_{\mathbb{Z}_+}(\mathbb{R}^n; \mathbb{R}^n)$ and $\{w_k\}_{k\geq 0}$ is an independent and identically distributed (i.i.d.) sequence with zero mean and unit variance. In this work, assume that w_k obeys Gaussian distribution $\mathcal{N}(0, 1)$ (see, e.g., [6,2,3,25,12]). This paper is to study almost sure stability of system (1) and its applications to stabilization problems of (linear) discrete-time stochastic systems.

In this paper, we apply some results of L-mixing processes [26] and define

$$\mathcal{F}_t = \sigma\{w_k : 0 \leq k \leq t\} \quad \text{and} \quad \mathcal{F}_t^+ = \sigma\{w_k : k \geq t+1\}. \tag{2}$$

It should be pointed out that one can employ some other techniques (see, e.g., [27]), instead of the L-mixing processes, in this work. Obviously, x_k is $\mathcal{F}_k$ measurable with respect to $(\mathcal{F}_k, \mathcal{F}_k^+)$ for all $k \geq 0$. Assume that f and g of class $C_{\mathbb{Z}_+}(\mathbb{R}^n; \mathbb{R}^n)$ satisfy the following conditions:

Assumption 2.1. For all $k \geq 0$, $f(0, k) \equiv g(0, k) \equiv 0$.

Assumption 2.2. For all $k \geq 0$, there is a function $\beta_L : \mathbb{R}_+ \to \mathbb{R}_+$ with $\beta_L(|x|) > 0$ for all $|x| > 0$ such that

$$|f(x, k)|^2 \vee |g(x, k)|^2 \geq \beta_L(|x|). \tag{3}$$

By virtue of continuity of functions f and g, there exists a unique adapted process $\{x_k\}_{k\geq 0}$ a.s., which is the solution of (1). Obviously, under Assumption 2.1, $\lim_{k\to\infty} x_k = 0$ a.s. on $\{\vartheta_0^{x_0} < \infty\}$, where

$$\vartheta_0^{x_0} = \inf\{t \geq 0 : |x_t| = 0\}. \tag{4}$$

Naturally, it is more interesting and important to study the (asymptotic) properties of the samples on $\{\vartheta_0^{x_0} = \infty\}$. The result given below describes a property of the solutions of (1) about $\vartheta_0^{x_0}$.

Proposition 2.1. *Suppose that f and g obey Assumptions 2.1–2.2. Then there exists a unique adapted process $\{x_k\}_{k\geq 0}$, which is the solution of (1), such that $\vartheta_0^{x_0} = \infty$ a.s.*

Proof. System (1) gives

$$|x_{k+1}|^2 = |f(x_k, k)|^2 + |g(x_k, k)|^2|w_{k+1}|^2 + 2f^T(x_k, k)g(x_k, k)w_{k+1}$$

for all $k \geq 0$, where $\{w_k\}$ is an i.i.d. sequence with $w_k \sim \mathcal{N}(0, 1)$. This with Assumption 2.2 implies $\mathbb{P}\{x_{k+1} = 0 | x_k \neq 0\} = 0$. Since $x_0 \neq 0$, this, by induction, gives $\mathbb{P}\{x_k = 0\} = 0$ for $k \geq 1$, which yields the desired result. □

Example 2.1. The scalar nonlinear system

$$\begin{aligned} x_{k+1} &= e^{\sin(k)}[x^{1/3}(k) + x(k)] \\ &\quad + [|x(k)|^{1/2} + e^{\cos(k)}x(k)]w_{k+1}, \quad x_0 \neq 0 \end{aligned} \tag{5}$$

satisfies Assumptions 2.1–2.2 and therefore obeys $\vartheta_0^{x_0} = \infty$ a.s.

The definition of L-mixing processes [26] is cited as follows, which is useful for the development of this work.

Definition 2.1. A stochastic process $\{s_n\}$ is L-mixing with respect to the σ-algebras $(\mathcal{F}_n, \mathcal{F}_n^+)$ if the following conditions are satisfied: (i) s_n is $\mathcal{F}_n$ measurable, (ii) $\sup_{n\geq 0} \mathbb{E}^{1/q}\left[|s_n|^q\right] < \infty$ for all $1 \leq q < \infty$, (iii) $\sum_{\tau=0}^{\infty} \gamma_q(\tau) < \infty$ for all $1 \leq q < \infty$, where

$$\gamma_q(\tau) = \sup_{n\geq\tau} \mathbb{E}^{1/q}\left[\left|s_n - \mathbb{E}[s_n|\mathcal{F}_{n-\tau}^+]\right|^q\right], \quad \tau \geq 0.$$

3. Almost sure stability of discrete-time stochastic systems

In this section, we study almost sure stability of discrete-time stochastic system (1) and establish a stability criterion based on

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