Systems & Control Letters 82 (2015) 40–50

Contents lists available at ScienceDirect

Systems & Control Letters

journal homepage: www.elsevier.com/locate/sysconle

An iterative method for suboptimal control of linear time-delayed systems

Seyed Mehdi Mirhosseini-Alizamini [a], Sohrab Effati [b,c,*], Aghileh Heydari [a]

[a] Department of Mathematics, Payame Noor University (PNU), P.O. Box 19395-3697, Tehran, Iran

[b] Center of Excellence on Soft Computing and Intelligent Information Processing (SCIIP), Ferdowsi University of Mashhad, Iran

[c] Department of Applied Mathematics, Faculty of Mathematical Sciences, Ferdowsi University of Mashhad, Iran

ARTICLE INFO

Article history:
Received 23 July 2014
Received in revised form
20 April 2015
Accepted 29 April 2015

Keywords:
Time-delay systems
Pontryagin's maximum principle
Variational iteration method
He's polynomials
Suboptimal control

ABSTRACT

This article presents a new approach for solving the Optimal Control Problem (OCP) of linear time-delay systems with a quadratic cost functional. The proposed method can also be used for designing optimal control time-delay systems with disturbance. In this study, the Variational Iteration Method (VIM) is employed to convert the original Time-Delay Optimal Control Problem (TDOCP) into a sequence of nonhomogeneous linear two-point boundary value problems (TPBVPs). The optimal control law obtained consists of an accurate linear feedback term and a nonlinear compensation term which is the limit of an adjoint vector sequence. The feedback term is determined by solving Riccati matrix differential equation. By using the finite-step iteration of a nonlinear compensation sequence, we can obtain a suboptimal control law. Finally, Illustrative examples are included to demonstrate the validity and applicability of the technique.

© 2015 Elsevier B.V. All rights reserved.

1. Introduction

The control of systems with time-delay has been of considerable concern. Delays occur frequently in biological, chemical, electronic and transportation systems [1]. Time-delay systems are therefore a very important class of systems whose control and optimization have been of interest to many investigators. The application of Pontryagin's maximum principle to the optimization of control systems with time-delays as outlined by Kharatishvili [2], results in a system of coupled TPBVP involving both delay and advance terms whose exact solution, except in very special cases, is very difficult. Therefore, the main object of all computational aspects of optimal time-delay systems has been to devise a methodology to avoid the solution of the mentioned TPBVP.

Concerning the solution of TDOCPs, Inoue et al. [3] have proposed a sensitivity approaches to optimization of linear systems with time delay. They expanded the control in Maclaurin's series in the delay and obtained the series coefficients from the solution of simple TPBVPs. The method presented in Jamshidi and Razzaghi [4], Malek-Zavarei [5] are also sensitivity approaches in which the original system is embedded in a class of non-delay systems using an appropriate parameter. Chen et al. [6] used the Taylor expansion to estimate time-delay state variables under the framework of Iterative Dynamic Programming (IDP). In general the computed response of the delay systems via the Taylor series is not in good agreement with the exact response of the system. The TDOCPs have also been solved using orthogonal functions. There are three classes of sets of orthogonal functions that are widely used. The first includes sets of piecewise constant basis functions (such as the Walsh functions, block pulse functions). The second consists of sets of orthogonal polynomials (such as the Legendre polynomials and Chebyshev polynomials). The third is the widely used sets of sine–cosine functions in Fourier series. This technique consists of reducing the problem to solve a system of algebraic equations. The approach is based on converting the underlying differential equation into integral equation through integration, approximating various signals involved in the equation by truncated orthogonal series, and using the operational matrix of integration to eliminate the integral operations. Typical examples are the Walsh functions [7], Chebyshev polynomials [8], and the linear Legendre multi-wavelets [9]. Recently, the TDOCPs have been solved using the hybrid functions, which consist of the combination of block-pulse functions with Legendre polynomials [10], Bernoulli polynomials [11], and Taylor polynomials [12].

In recent years, a growing interest has been appeared toward the application of VIM techniques in various types of problems,

* Corresponding author at: Center of Excellence on Soft Computing and Intelligent Information Processing (SCIIP), Ferdowsi University of Mashhad, Iran.
E-mail addresses: seyedmehdi_mirhosseini@yahoo.com (S.M. Mirhosseini-Alizamini), s-effati@um.ac.ir (S. Effati).

http://dx.doi.org/10.1016/j.sysconle.2015.04.013
0167-6911/© 2015 Elsevier B.V. All rights reserved.

and many new methods have been introduced in the literature. The variational iteration method, which was proposed originally by the Chinese mathematician He [13–15], has been proved by many authors as a powerful mathematical tool for various kinds of linear and nonlinear problems [16–20]. The reliability of the method and the reduction in the size of computational work gave this method wider applications. The main idea in the VIM is to construct an iterative sequence of functions converging to an exact solution. Since the method works without discretization, linearization, transformation, or perturbation of the problem, it is not affected by round of error. The VIM has been applied successfully in a wide range of problems, such as partial differential equations [21], fractional differential equations [22],delay differential equations [14], and integro-differential equations [23]. The convergence of He's VIM has been discussed in [24,25]. The variational iteration method using He's polynomials has been introduced in [26–28]. Among a various number of VIM applications, the use of this method in solving Riccati equations (see for example [29]), made it a powerful tool in the context of control theory. However, for time delay optimal control problems, the VIM is still not applied.

The aim of this paper is to employ the VIM for solving the OCP of linear time-delay systems with a quadratic cost functional. In addition, the proposed method can also be used for a class of linear time-delay systems affected by external persistent disturbance. Applying the VIM, the original TDOCP is transformed into a sequence of linear TPBVPs involving both delay and advance terms. The optimal control law obtained consists of an accurate linear feedback term and a nonlinear compensation term which is the limit of an adjoint vector sequence. By using the finite-step iteration of the nonlinear compensation sequence, we can obtain a suboptimal control law.

The paper is organized as follows. Section 2 is devoted to Pontryagin's maximum principle used for solving linear time-delay optimal control problem. Section 3 is dedicated to the proposed design approach to solve a close-loop optimal control problem based on the VIM, modified variational iteration method (MVIM) and convergence of the method is demonstrated. We present the model of the system with an external persistent disturbance in Section 4. Section 5 explains how to use the results of Sections 3 and 4 in practice. In this section, in order to obtain a suboptimal control law, an efficient algorithm with low computational complexity and fast convergence rate is presented. In Section 6 the numerical examples are simulated to show the resemblance of our theory and demonstrate the performance of our network. Finally, we end this paper with conclusions in Section 7.

2. Problem statement and optimality conditions

First, let us introduce a definition.

Definition 2.1. A function $x(t)$ from an interval $[t_0, t_f]$ of the real numbers into an Euclidean space $\mathbb{R}^n$ is called piecewise continuous on this interval except for some finite number of jump discontinuities. We denote the vector space of such functions by $PC([t_0, t_f], \mathbb{R}^n)$, and its subspace whose elements are continuous and have piecewise continuous first order derivatives by $PC^1([t_0, t_f], \mathbb{R}^n)$ [30].

Consider a linear system with state time-delay described by:

$$\begin{cases} \dot{x}(t) = Ax(t) + A_1x(t-\tau) + Bu(t), & t_0 \leqslant t \leqslant t_f, \\ x(t) = \phi(t), & t_0 - \tau \leqslant t \leqslant t_0, \end{cases} \tag{2.1}$$

where $x(t) \in PC^1([t_0 - \tau, t_f], \mathbb{R}^n)$ and $u(t) \in PC([t_0, t_f], \mathbb{R}^m)$, are the state and control vectors respectively, $\tau > 0$ is the constant time-delay, A, A_1 and B are real constant matrices of appropriate dimensions, and $\phi(t) \in C([t_0 - \tau, t_0], \mathbb{R}^n)$ is the continuous initial state function. Here, it is assumed that the pair $(A + A_1, B)$ is controllable [30]. The objective is to find the optimal control law $u^*(t)$ over $t \in [t_0, t_f]$, which minimizes the following quadratic cost functional subject to the system (2.1):

$$J = \frac{1}{2}x^T(t_f)Q_f x(t_f) + \frac{1}{2}\int_{t_0}^{t_f} \left(x^T(t)Qx(t) + u^T(t)Ru(t)\right) dt, \tag{2.2}$$

where $Q_f \in \mathbb{R}^{n\times n}$ and $Q \in \mathbb{R}^{n\times n}$ are positive semi-definite matrices, and $R \in \mathbb{R}^{m\times m}$ is a positive definite matrix.

Since the performance index (2.2) is convex, the following extreme necessary conditions are also sufficient for optimality [2,13,30]:

$$\begin{cases} \dot{x} = H_\lambda(x, u^*, \lambda, t), \\ \dot{\lambda} = -H_x(x, u^*, \lambda, t), \\ u^* = \operatorname{argmin}_u H(x, u, \lambda, t) \\ x(t) = \phi(t), \quad t_0 - \tau \leqslant t \leqslant t_0, \end{cases} \tag{2.3}$$

where

$$H(x, u, \lambda, t) = \frac{1}{2}x^T(t)Qx(t) + \frac{1}{2}u^T(t)Ru(t) + \lambda^T(t)[Ax(t) + A_1x(t-\tau) + Bu(t)]. \tag{2.4}$$

According to Pontryagin's maximum principle of OCPs with time-delay (2.1), the necessary conditions of optimality can be written as:

$$\begin{cases} \dot{x}(t) = Ax(t) + A_1x(t-\tau) - S\lambda(t), & t_0 \leqslant t \leqslant t_f, \\ \dot{\lambda}(t) = \begin{cases} -Qx(t) - A^T\lambda(t) - A_1^T\lambda(t+\tau), \\ \quad t_0 \leqslant t < t_f - \tau, \\ -Qx(t) - A^T\lambda(t), \quad t_f - \tau \leqslant t \leqslant t_f, \end{cases} \\ x(t) = \phi(t), \quad t_0 - \tau \leq t \leqslant t_0, \\ \lambda(t_f) = Q_f x(t_f), \end{cases} \tag{2.5}$$

where $S = BR^{-1}B^T$, $x(t-\tau)$ is time-delay term and $\lambda(t+\tau)$ is time-advance term, furthermore $\lambda(t) \in PC^1([t_0, t_f], \mathbb{R}^n)$ is the co-state vector. Also, the optimal control law is given by:

$$u^*(t) = -R^{-1}B^T\lambda(t), \quad t_0 \leqslant t \leqslant t_f. \tag{2.6}$$

The optimal can be implemented as a closed loop optimal if the co-state vector obtained consists of linear function of the states and a nonlinear term which is the adjoint vector sequence, in the form

$$\lambda(t) = P(t)x(t) + g(t), \qquad \lambda(t_f) = Q_f x(t_f), \tag{2.7}$$

where $P(t) \in \mathbb{R}^{n\times n}$ is unknown positive-semidefinite function matrix, $g(t) \in \mathbb{R}^n$ is the adjoint vector.

Calculating the derivatives of both sides of Eq. (2.7), we get

$$\begin{aligned} \dot{\lambda}(t) &= \dot{P}(t)x(t) + P(t)\dot{x}(t) + \dot{g}(t), \quad t_0 \leqslant t \leqslant t_f \\ &= \left[\dot{P}(t) + P(t)A - P(t)SP(t)\right]x(t) - P(t)Sg(t) \\ &\quad + P(t)A_1x(t-\tau) + \dot{g}(t), \end{aligned} \tag{2.8}$$

on the other hand, substituting (2.7) in the necessary conditions (2.5), we have

$$\dot{\lambda}(t) = \begin{cases} -Qx(t) - A^TP(t)x(t) - A^Tg(t) \\ \quad - A_1^TP(t+\tau)x(t+\tau) - A_1^Tg(t+\tau), \\ \quad t_0 \leqslant t < t_f - \tau, \\ -Qx(t) - A^TP(t)x(t) - A^Tg(t), \quad t_f - \tau \leqslant t \leqslant t_f. \end{cases} \tag{2.9}$$

Download English Version:

https://daneshyari.com/en/article/7151744

Download Persian Version:

https://daneshyari.com/article/7151744

Daneshyari.com