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Input delay compensation of linear systems with both state and input delays by adding integrators☆

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ABSTRACT

This paper studies stabilization of linear systems with both state and input delays. A dynamic input-delay compensator obtained by adding integrators is established to compensate the input delays that can be arbitrarily large. With the input delay compensator, the original stabilization problem reduces to the problem of stabilizing an augmented linear time-delay system without input delay. Three methods are also proposed to design stabilizing controllers for the augmented linear time-delay system. The first method is based on linear matrix inequalities (LMIs) and the second method is based on model reduction. The third method is based on pole placement and is built for the particular case that the original time-delay system has only a pure delayed state vector on its right hand side. For this method, the optimal gain such that the decay rate of the closed-loop system is maximized is also proposed. The effectiveness of the proposed approaches is illustrated by three linear time-delay systems that are open-loop unstable.

1. Introduction

Time delay systems have received considerable attentions during the past several decades because of their wide applications in engineering and their infinite dimensional behavior that is theoretically challenging [1–5]. For systems with delays existing only in the actuators, the predictor-type (also known as model reduction) controllers have been extensively investigated in the literature [6–9]. The advantages of the predictor-type controllers are that the closed-loop systems only possess finite spectrum and arbitrary decay rates can be assigned. The main difficulty of this approach is that the resulting controllers involve integrals of the past controls and thus can only be implemented by numerical integrals in most cases [9]. Very careful attention should be put on such approximation since it may result in instability of the overall closed-loop systems [10].

The classical predictor-type controllers have now been extended to linear systems that have both state and input delays. In [11], a general system in strict-feedback form with delayed integrators is investigated and a predictor-type feedback controller is proposed. In [12], delay-adaptive predictor feedback control for linear feedforward systems is studied. In [13], the forwarding and backstepping approaches built initially in nonlinear control are extended to the design of controllers achieving finite spectrum assignment for linear systems that have both state and input delays and possess some feedforward and feedback structures. The classical predictor-type controllers have also been extended to nonlinear systems that have both state and input delays [14,15]. All of these methods mentioned above lead to static and/or dynamic controllers that involve integrals of the states and/or the inputs. Recently, for a class of linear systems with both (time-varying) state and input delays, and all eigenvalues being at zero (which is a generalization of multiple integrators), linear controllers using only the current state (namely, $x(t)$) are designed in [16] to achieve stability.

In this paper we study stabilization of linear systems with both state and input delays that are constant and can be arbitrarily large. To solve the underlying difficulty that the input delays may be (significantly) larger than the state delay, a dynamic compensator obtained by adding integrators is established to compensate the large input delays. With the proposed input-delay compensator, the original stabilization problem reduces to the problem of stabilizing an augmented linear time-delay system without input delay (or the input delay equaling the state delay). Three methods are then developed to stabilize the augmented time-delay system. The first method is based on LMIs and the second method is based on model reduction which relies on solutions to a class of nonlinear

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matrix equations. By assuming that the nonlinear matrix equation associated with the original time-delay system in the absence of input delay is available, solutions to the nonlinear matrix equation associated with the augmented time-delay system can be obtained immediately by solving a linear matrix equation. The third stabilizing controller design method is established for the case that the term $x(t)$, which is the state vector, does not appear in the system equation. This method is based on the fact that the linear time-delay system $\dot{x}(t) = Ax(t-\tau)$ is asymptotically stable if and only if the poles of A are located within some bounded region referred as stable region. As a result, stabilization of the augmented linear time-delay system can be achieved if controllers are designed such that the closed-loop system takes the form $\dot{x}(t) = Ax(t-\tau)$ and the poles of A are within the stable region. For this method, we also discuss how to design the gains of the controllers such that the decay rate of the closed-loop system is maximized. Three numerical examples are given to illustrate the effectiveness of the proposed approaches.

The problem of compensating input delay for linear systems with both input and state delays, as far as we know, was studied in [17] for the first time. The approach in [17] is based on nested predictor which results in controllers involving multiple integrals. This paper provides an alternative solution to the input delay compensation problem by adding integrators. The resulting controller is a dynamic compensator. The dynamic compensator possesses some advantages over the nested predictor based compensator. On the one hand, the dynamic compensator is in an explicit and simple form, while the nested predictor based compensator is defined in a recursive way and is quite complicated as it involves multiple integrals. On the other hand, the dynamic compensator is more easy to implement than the nested predictor based compensator as the later one involves both input filters and multiple integrals; yet their control performances are comparable. This fact will be approved by a numerical example in Section 5.2.

The remainder of this paper is organized as follows. The general idea of input delay compensation by adding integrators will be introduced in Section 2. The model reduction approach is then built in Section 3 to stabilize the augmented time-delay system. In the case that the state vector $x(t)$ does not appear in the system equation, a pole placement approach will be established in Section 4 to stabilize the augmented time-delay systems obtained by adding integrators. Examples are given in Section 5 and, finally Section 6 concludes the paper.

2. Input delay compensation by adding integrators

Consider a time-delay system with both state and input delays in the form of

$$\dot{x}(t) = A_0 x(t) + A_\tau x(t-\tau) + B_h u(t-h), \quad t \geq 0, \tag{1}$$

where $h = (\delta-1)\tau + \Delta\tau$ with $\delta \geq 1$ being an integer and $\Delta\tau \in [0,\tau)$, and $(A_0, A_\tau, B_h) \in \left(\mathbf{R}^{n\times n}, \mathbf{R}^{n\times n}, \mathbf{R}^{n\times m}\right)$ are known constants. Let the initial condition of the above system be $x(\theta), u(\theta), \forall\theta \in [-\delta\tau, 0]$. In this paper, we consider the stabilization problem for system (1). The difficulty of this problem lies in the fact that the delay appears in both the state and the input and, moreover, the delays can be arbitrarily large (but bounded). If $A_\tau = 0$, the delay system (1) can be efficiently controlled by the classical predictor feedback which aims to compensate the input delay so that the resulting closed-loop system is delay-free (see, for example, [6]). The classical predictor feedback approach was recently generalized in [17] to stabilize system (1) by assuming that a stabilizing controller is available for the corresponding time-delay system

$$\dot{x}(t) = A_0 x(t) + A_\tau x(t-\tau) + B_h u(t), \quad t \geq 0, \tag{2}$$

which does not have input delay. The idea in [17] is to compensate the input delay of system (1) by using nested predictors. The resulting controller involves multiple integrals and must be implemented by approximated numerical integration [17]. In this paper, we will provide an alternative input delay compensation approach for system (1) by adding integrators.

The idea of "adding integrators" can be explained as follows. Firstly, by setting

$$u(t) = \bar{u}(t - (\tau - \Delta\tau)), \tag{3}$$

the delay system (1) can be expressed as

$$\dot{x}(t) = A_0 x(t) + A_\tau x(t-\tau) + B_h \bar{u}(t-\delta\tau). \tag{4}$$

Now we choose an auxiliary state vector $Z_1(t)$ as

$$Z_1(t) = \bar{u}(t - (\delta-1)\tau). \tag{5}$$

Then the time-delay system in (4) can be rewritten as

$$\dot{x}(t) = A_0 x(t) + A_\tau x(t-\tau) + B_h Z_1(t-\tau). \tag{6}$$

It follows from (5) that $\bar{u}(s) = Z_1(s)\mathrm{e}^{(\delta-1)\tau s}$ by which we see that $\bar{u}(t)$ needs to know $(\delta-1)\tau$ seconds ahead of $Z_1(t)$. We then consider the integrator

$$\dot{Z}_1(t) = Z_2(t-\tau), \tag{7}$$

which implies $\bar{u}(s) = \frac{1}{s}Z_2(s)\mathrm{e}^{(\delta-2)\tau s}$, namely, $\bar{u}(t)$ needs to know only $(\delta-2)\tau$ seconds ahead of $Z_2(t)$. Repeat the above process by adding the integrators

$$\dot{Z}_i(t) = Z_{i+1}(t-\tau), \quad i \in \mathbf{I}[2, \delta-1] \triangleq \{2, 3, \ldots, \delta-1\}, \tag{8}$$

to give $\bar{u}(s) = \frac{1}{s^{\delta-1}}Z_\delta(s)$. Hence $\bar{u}(t)$ can be obtained by integrating $Z_\delta(t)$ without knowing the future information of $Z_\delta(t)$. Finally, we choose the (new control) vector $Z_{\delta+1}(t)$ such that

$$\dot{Z}_\delta(t) = Z_{\delta+1}(t), \quad t \geq 0, \tag{9}$$

and it follows that $\bar{u}(s) = \frac{1}{s^\delta}Z_{\delta+1}(s)$. Now we rewrite the overall system consisting of (6)–(9) as

$$\begin{cases} \dot{x}(t) = A_0 x(t) + A_\tau x(t-\tau) + B_h Z_1(t-\tau), \\ \dot{Z}_i(t) = Z_{i+1}(t-\tau), \quad i \in \mathbf{I}[1, \delta-1], \\ \dot{Z}_\delta(t) = Z_{\delta+1}(t), \end{cases} \tag{10}$$

or equivalently,

$$\dot{\zeta}(t) = \mathscr{A}_0\zeta(t) + \mathscr{A}_\tau\zeta(t-\tau) + \mathscr{B}Z_{\delta+1}(t), \tag{11}$$

where $\zeta(t) = \left[x^{\mathrm{T}}(t), Z_1^{\mathrm{T}}(t), \ldots, Z_\delta^{\mathrm{T}}(t)\right]^{\mathrm{T}} \in \mathbf{R}^{n+m\delta}$ is the augmented state vector, $Z_{\delta+1}(t)$ is the input vector to be designed, and

$$\mathscr{A}_0 = \begin{bmatrix} A_0 & 0_{n\times m\delta} \\ 0_{m\delta\times n} & 0_{m\delta\times m\delta} \end{bmatrix},$$

$$\mathscr{A}_\tau = \begin{bmatrix} \begin{bmatrix} A_\tau & B_h \end{bmatrix} & 0_{n\times m(\delta-1)} \\ 0_{m(\delta-1)\times(n+m)} & I_{m(\delta-1)} \\ 0_{m\times(n+m)} & 0_{m\times m(\delta-1)} \end{bmatrix}, \tag{12}$$

$$\mathscr{B} = \begin{bmatrix} 0_{(n+(\delta-1)m)\times m} \\ I_m \end{bmatrix}.$$

Notice that the time-delay system in (11) has only a single state delay τ which is the same as that in the original system (1), and has *no input delay*.

By adding integrators, stabilization of the original time-delay system (1) with both state and input delays now reduces to the stabilization of the augmented time-delay system (11) without input delay, namely, the input delay is "compensated". In the following, we show how to use stabilizing controllers for the augmented time-delay system (11) to stabilize the original time-delay system (1).