Systems & Control Letters 73 (2014) 58–66

Contents lists available at ScienceDirect

Systems & Control Letters

journal homepage: www.elsevier.com/locate/sysconle

Consensus control of nonlinear leader–follower multi-agent systems with actuating disturbances[☆]

Xinghu Wang [a], Dabo Xu [b,*], Yiguang Hong [a]

[a] Key Laboratory of Systems and Control, Institute of Systems Science, Chinese Academy of Sciences, Beijing 100190, China

[b] School of Automation, Nanjing University of Science & Technology, Nanjing 210094, China

ARTICLE INFO

Article history:
Received 8 February 2014
Received in revised form
3 July 2014
Accepted 9 September 2014

Keywords:
Multi-agent systems
Disturbances
Distributed control
Networked control

ABSTRACT

This paper studies a semi-global asymptotic consensus problem of nonlinear multi-agent systems with local actuating disturbances. For a modest nonlinear scenario, a consensus protocol is proposed based on a viable two-layer network. The consensus problem is treated as distributed output regulation, which is resolved by a joint decomposition of the zero-error constraint inputs and a configuration of a flexible internal model network. An illustrative example is also given to show the efficiency of the two-layer networked design.

1. Introduction

In the past few years, cooperative control of multi-agent systems has gained increasing research interest and a major development. One of the central problems is that of controlling all agents in order to make their outputs converge to a common output trajectory. This problem usually refers to as consensus or output synchronization. Since the fundamental study of consensus protocols for single-integrator agents (see [1]), a number of effective techniques have been proposed for multi-agent systems from linear to nonlinear in many directions, see for instances [2–7] and references therein. At the present stage, to the best of our knowledge, there is a lack of particular studies on consensus control with external disturbances appearing at individual agent dynamics. Relevant results may be found in [8–10]. In particular, [9,10] proposed an observer-based control by viewing the disturbances as exosystem outputs and [8] studied the problem by a non-smooth control technique. It is noted that the methods developed in the aforementioned literature rely on the absolute state information in addition to the neighbor information. Also, the result of [8] is a practical consensus design, not leading to asymptotic consensus. In view of these existing studies, consensus control with exogenous disturbances deserves further investigation, particularly for nonlinear multi-agent networks.

It is known that, to cope with asymptotic tracking and/or disturbance rejection of uncertain systems, in the terminology of robust output regulation, the device of internal model plays an indispensable role, see [11–16]. Recently, a lot of efforts have been made to distributed control of the leader–follower multi-agent systems with uncertainties by applying the output regulation theory; see, for instances [17–19]. In accordance with these results, an individual internal model should be embedded in each local controller to succeed the consensus with node uncertainties and disturbances. Recall that in output regulation, the exosystem is often used to model references and disturbances as well. In the usual centralized or decentralized setup, there is no need to treat them separately. Nonetheless, regarding multi-agent systems in the distributed fashion, things are basically inconsistent due to limited interactions or communications. To adapt this situation, the leader in leader–follower type networks can be viewed as an exosystem to produce references relating to certain collective behaviors. Contrary to references, local disturbances certainly have a negative effect to the control goal. These two types of signals thus have opposite effects with respect to the control goal. This fact basically motivates us to develop more flexible strategies that can manage

☆ The work was substantially supported by NNSF of China under Grant 61304009 and Grant 61174071, and under A Project Funded by the Priority Academic Program Development of Jiangsu Higher Education Institutions and the Program for Changjiang Scholars and Innovative Research Team in University (No. IRT13072). A preliminary version of the paper was presented at the 11th IEEE International Conference on Control & Automation [31].

* Corresponding author. Tel.: +86 25 8431 5872.
E-mail addresses: wxh@amss.ac.cn (X. Wang), dabo.xu@ymail.com, xu.dabo@gmail.com (D. Xu), yghong@iss.ac.cn (Y. Hong).

http://dx.doi.org/10.1016/j.sysconle.2014.09.004
0167-6911/© 2014 Elsevier B.V. All rights reserved.

them in some separable and general manner to realize the necessary input compensation for all controlled agents.

In the paper, we study an asymptotic consensus design for a class of locally disturbed homogeneous multi-agent systems; refer to [20,21] for consensus control of homogeneous networks in the absence of disturbances. Our main objective is to demonstrate how a networked internal model can be implemented in a more general network to solve the problem. In the problem, each controller is required to asymptotically reject its local disturbances, meanwhile to reach consensus as chief control goal. To do the task, we employ a novel configurable internal model network, distinguished from the output-interaction network. It is shown that some agents only need an internal model to reject its local disturbance. The new treatment still allows us to engage a scheme of converting the consensus problem into a tractable distributed stabilization problem of an augmented network. For an arbitrary initial region, the latter stabilization problem is further solved by a linear high-gain type distributed stabilizer.

The proposed design belongs to two-layer networked control in a broadened level than the usual distributed consensus control. Particular attentions will be paid to the links among local controllers. A relevant two-layer networked control was studied in [22]. Regarding the consensus protocol developed in the present study, three main reasons are summarized to explain why we do so. First, in most existing results (e.g. [17–19]), the internal model communication has not been carefully considered, especially when communication is "cheap". Our study at least leads to an interesting alternative. Second, by the internal model based consensus control, the controller network itself has a certain "consensus" property, since it contains internal models that are essentially "copies" to the controlled agents. This observation actually has evoked the interest of building up links among local controllers for the purpose of sharing useful information. Third, from the viewpoint of computing the consensus protocol, reducing its complexity certainly makes sense. In summary, our study can provide flexible strategies especially when control complexity matters and communication is comforted.

The rest of this paper is organized as follows. In Section 2, some preliminaries are given. In Section 3, the configurable internal model network is elaborated and the main result is presented. In Section 4, an example is given for an illustration. Finally in Section 5, the paper is closed with some concluding remarks. The graph notations used throughout the paper are put in the Appendix.

Notations: throughout the paper, for any column vectors $x_1, \ldots, x_n$, $(x_1, \ldots, x_n)$ denotes $\left[x_1^\top, \ldots, x_n^\top\right]^\top$ if no confusion arises in the context; for a real number $\rho > 0$, $\mathcal{B}^x_\rho = \{x \in \mathbb{R}^n : \|x\| \le \rho\}$; for a real number $c > 0$ and a smooth positive definite and radially unbounded function $W : \mathbb{R}^n \to \mathbb{R}$, $\Omega_c(W) := \{x \in \mathbb{R}^n : W(x) \le c\}$.

2. Preliminaries

The focus of this paper is on leader-following consensus of nonlinear multi-agent systems with local disturbances, consisting of a leader and a set of controlled nonlinear uncertain systems. The leader with node index 0 is described by

$$\dot{v}_r = S_r v_r, \qquad y_0 = q(v_r, w) \tag{1}$$

where $v_r \in \mathbb{R}^{n_{v_r}}$ is the state and $y_0 \in \mathbb{R}$ is the desirable reference characterizing the collective output behavior. The follower agents are assumed to be globally transformable into the form

$$i \in \mathcal{O} : \quad \begin{cases} \dot{z}_i = f(z_i, y_i, w) \\ \dot{y}_i = g(z_i, y_i, w) + \delta_i(v_i, w) + u_i \end{cases} \tag{2}$$

where $\mathcal{O} := \{1, \ldots, N\}$ denotes the follower node set and for each agent $i \in \mathcal{O}$, $(z_i, y_i) \in \mathbb{R}^n$ is the state, $y_i \in \mathbb{R}$ is the output, $u_i \in \mathbb{R}$ is the control input, $w \in \mathbb{W}$ is the parameter uncertainty in a known compact set $\mathbb{W} \subset \mathbb{R}^{n_w}$ and $\delta_i(v_i, w)$ is the local actuating disturbance of agent i with v_i governed by some local disturbance source

$$\dot{v}_i = S_i v_i, \quad v_i \in \mathbb{R}^{n_{v_i}}. \tag{3}$$

It is noticed that the local disturbance sources and the leader are divided, different from the usual. We also assume that each of the matrices S_r and S_i, $i = 1, \ldots, N$ has distinct eigenvalues lying on the imaginary axis such that (1) and (3) can generate the fundamental sinusoidal/step type signals; cf. *Assumption* A2.2 in [14] and *Assumption* 3 in [16]. This type of disturbances have been studied in the framework of output regulation, see, for instances, [11,15]. The functions f, g, q and δ_i are polynomials in their arguments, satisfying

$f(0, 0, w) = 0, g(0, 0, w) = 0, q(0, w) = 0, \delta_i(0, w) = 0, \forall w \in \mathbb{W}.$

We denote $z := (z_1, \ldots, z_N) \in \mathbb{R}^{N(n-1)}$, $y := (y_1, \ldots, y_N) \in \mathbb{R}^N$.

Remark 1. The follower agent in (2) is called a strict-feedback uncertain nonlinear system having unity relative degree [23,24] that is basic and popular in nonlinear control, see, e.g. [25,17,18,26]. The consensus of this class of nonlinear multi-agent systems can cover interesting synchronization problems of a number of benchmark nonlinear oscillators, including Lorenz systems, FitzHugh–Nagumo (FHN) systems (that will be discussed in Section 4), etc., see [26].

Let $v = (v_r, v_1, \ldots, v_N)$ with initial condition $v(0)$ starting in a known compact region $\mathbb{V}$. Clearly, we have another compact set $\mathbb{V}'$ such that for any $v(0) \in \mathbb{V}$, its response $v(t) := v(t, v(0))$ satisfies $v(t) \in \mathbb{V}'$ for all $t \ge 0$. It is denoted that $\mathbb{D} := \mathbb{V}' \times \mathbb{W}$. Denote the regulated output $e = (e_1, \ldots, e_N)$ where

$$i \in \mathcal{O} : \quad e_i = y_i - y_0$$

which is usually unavailable to every follower agent. In practice, determined by an output-interaction graph $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$ with $\mathcal{V} = \{0, 1, 2, \ldots, N\}$ (turn to Appendix at the end of the paper for graph notations), agent i has a neighbor-based measurement e_{mi} as follows:

$$i \in \mathcal{O} : \quad e_{mi} = \sum_{j \in \mathcal{V}} a_{ij}(y_i - y_j) = \sum_{j \in \mathcal{V}} a_{ij}(e_i - e_j) \tag{4}$$

with $e_0 := 0$.

Besides the graph $\mathcal{G}$ (referred to as network sometimes in the paper), another graph $\mathcal{G}^c = \{\mathcal{O}, \mathcal{E}^c\}$, to denote an internal model network, is configured to indicate the internal model communications, to be elaborated in Section 3.1. One of the main objectives of the paper is to implement a communication network in a more extended distributed control setup than the single-layer one where only $\mathcal{G}$ is considered.

Specifically, we study a semi-global leader-following consensus problem of the systems (1) and (2) in the presence of the disturbance (3), formulated as follows. *For any graph $\mathcal{G}^c$ (as the internal model network admitting certain physical requirements, if any) and for any compact sets $\mathcal{B}^z_\rho$ and $\mathcal{B}^y_\rho$, find a smooth distributed controller*[1]

$$i \in \mathcal{O} : \quad \begin{cases} \dot{\xi}_i = h_{\xi i}(\xi_i, \xi_j, u_i, j \in \mathcal{N}_i^c) \\ u_i = u_{ci}(\xi_i, \xi_j, e_{mi}, j \in \mathcal{N}_i^c) \end{cases} \tag{5}$$

together with a specified compact set $\mathcal{B}^{\xi'}_{\rho'}$, $\xi' := (\xi_1, \ldots, \xi_N)$, such that, for each $(v(0), w, z(0), y(0), \xi'(0)) \in \mathcal{B}'$, $\mathcal{B}' := \mathbb{V} \times \mathbb{W} \times \mathcal{B}^z_\rho \times \mathcal{B}^y_\rho \times \mathcal{B}^{\xi'}_{\rho'}$, both the following conditions hold

(i) *the trajectory of the closed-loop system composed of (1), (2) and (5) exists for all $t \ge 0$ and is bounded over $[0, +\infty)$;*

[1] $\mathcal{N}_i^c := \{j \in \mathcal{O} : (j, i) \in \mathcal{E}^c\}$ means the neighbor set of agent i in the graph $\mathcal{G}^c$.