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The disturbance decoupling problem with stability for switching dynamical systems

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ABSTRACT

The disturbance decoupling problem with stability is dealt with by means of the geometric approach for switching systems. The existence of feedbacks which decouple the disturbance and, at the same time, assure stability is difficult to characterize, since the action of the feedback couples with that of the switching law. Under suitable conditions, it is shown that the above requirement can be dealt with in separate ways and this allows us to state a checkable necessary condition and, on that basis, also a sufficient condition for solvability of the problem.

1. Introduction

The geometric approach developed by Basile and Marro [1] and by Wonham [2] in the early 1970 has been, since then, successfully applied to a large class of control problems in the field of linear dynamical systems. The approach has been then extended to other classes of dynamical systems, providing solutions to many noninteracting control problems, regulation problems and observation problems.

More recently, geometric concepts have been applied to the study of control problems which involve switching systems. In many control, regulation or observation problems, solutions are characterized by structural conditions, concerning existence and properties of subspaces of the state space, and by qualitative conditions, in general concerning stability of specific subsystems. The geometric approach has proven to be well suited for dealing with structural issues and, therefore, considering a switching system Σ_σ, it is quite natural to employ it in analyzing structural properties of the modes Σ_i, $i \in I$ that, in an appropriate sense, do not depend on the variation of the index $i \in I$. On the other hand, switching largely affects qualitative properties, like stability, and therefore the fulfillment of qualitative conditions is mainly related to properties of σ, both when σ can be appropriately chosen to control the system and when it cannot be chosen. Although it is not always possible to decouple structural properties of the modes from qualitative properties of the switching system, this approach provides valuable insight into many cases, where it gives complete or partial characterization of solutions to problems of above-mentioned kinds. Previous examples along this line are the results of [3–6] where the problem of decoupling a disturbance from the output of a switching system, under various conditions, has been considered, and those of [7–9] where a regulation problem for a switching system has been considered.

In this paper, we revise the formulation of the Disturbance Decoupling Problem with Stability (DDPS) by means of state feedback in the case in which the switching rule depends only on time (differently from [4,5], where state-dependent switching rules are considered) and can be conveniently chosen. The problem is studied by using geometric concepts and by introducing a new characterization of specific properties of time-dependent switching rules. The existence of solutions is characterized, under suitable hypothesis, by means of structural geometric conditions and of qualitative conditions that are not coupled, but that can be fulfilled independently one from the other. This makes possible to get, first, a necessary condition for the existence of solutions and, then, a sufficient condition that, when the necessary one is verified, is practically checkable and constructive. These results improve and deepen those already proved in [3].

The paper is organized as follows. In Section 2, we present and discuss the geometric notions of controlled invariance and conditioned invariance for switching systems. Structural decomposition with respect to controlled invariance is one of the fundamental concepts we introduce. The problem of decoupling a disturbance from the output by means of state feedback is discussed in Section 3. We analyze the case in which decoupling has to be

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achieved for arbitrary choice of the switching rule (Disturbance Decoupling Problem under Arbitrary Switching, or DDPAS) and then, adding the requirement of stability, we concentrate on the case in which both decoupling and stability have to be achieved by choosing, in addition to the state feedback, a specific switching rule. Motivations for taking into account this way of stating the problem are discussed in Remark 1. Specific properties of the switching rule, namely *exhaustiveness* and *essentiality*, are defined and studied in order to give, in Proposition 7, a first necessary condition for the solvability of the DDPS. In Section 4, the necessary condition for solvability is restated, under slightly more restrictive hypotheses, showing that stability does not depend on the choice of the state feedback which fulfills the structural requirements. The insight provided in this way makes possible to state, in Proposition 8, a necessary condition for solvability of the DDPS that exploits properties of switching systems whose modes are characterized by normal dynamic matrices. Section 5 contains conclusions and it outlines a possible way to investigate further and to ameliorate the sufficient condition.

2. Preliminaries

Let $\mathbb{R}$ and $\mathbb{R}^+$ denote respectively the field of real numbers and the subset of nonnegative ones. We consider the switching linear system Σ_σ defined by the equations

$$\Sigma_\sigma \equiv \begin{cases} \dot{x}(t) = A_{\sigma(t)}x(t) + B_{\sigma(t)}u(t) \\ y(t) = C_{\sigma(t)}x(t) \end{cases} \tag{1}$$

where $t \in \mathbb{R}^+$ is the time variable; $x \in \mathcal{X} = \mathbb{R}^n$ is the state; $u \in \mathcal{U} = \mathbb{R}^m$ is the input; $y \in \mathcal{Y} = \mathbb{R}^p$ is the output; σ is a function, representing a switching rule, which takes values in the set $I = \{1, \dots, N\}$ and that, in our framework, is assumed to depend on time only, that is $\sigma : \mathbb{R}^+ \to I$, and, finally, for any value $i \in I$ taken by σ, A_i, B_i, C_i are matrices of suitable dimensions with real entries. Without loss of generality, we will assume that B_i is full column rank for all $i \in I$.

In other terms, a switching system Σ_σ consists of an indexed family $\Sigma = \{\Sigma_i\}_{i\in I}$ of continuous-time, linear systems of the form

$$\Sigma_i \equiv \begin{cases} \dot{x}(t) = A_i x(t) + B_i u(t) \\ y(t) = C_i x(t) \end{cases} \quad \text{for } i = 1, \dots, N, \tag{2}$$

called modes of Σ_σ, and of a supervisory switching rule σ, whose value $\sigma(t)$ specifies the mode which is active at time t. A standard requirement on σ is that it generates only a finite number of switches on any time interval of finite length, so to exclude chattering phenomena. This will be guaranteed by asking that the length of the time interval between any two consecutive switchings is not smaller than a constant τ_σ, called the dwell-time of σ, with $\tau_\sigma > 0$. According to applications, interest can be in studying properties of Σ_σ which hold for any choice of the switching rule σ, as well as in investigating the existence of (restricted classes of) switching rules that guarantee the fulfillment of specific requirement (see e.g. [10]).

In the above-defined framework, we introduce a number of geometric notions and results for switching systems of form (1). Proofs of the statements in this Section can be obtained by applying standard geometric techniques in the same way as described in [1,2] in the corresponding situations or can be found in the quoted references.

2.1. Controlled invariance and conditioned invariance

Definition 1 (*[11,12]*)**.** Given a family $\Sigma = \{\Sigma_i\}_{i\in I}$ of linear systems of the form (2), a subspace $\mathcal{V} \subseteq \mathcal{X}$ is called a *robust controlled invariant subspace* for Σ, or a *robust* $(A_{i\in I},\ B_{i\in I})$*-invariant subspace*, if $A_i\mathcal{V} \subseteq \mathcal{V} + \operatorname{Im} B_i$ for all $i = 1, \dots, N$. If Σ_σ is a switching linear system of form (1) defined by the elements of Σ, any robust controlled invariant subspace $\mathcal{V}$ for Σ is said to be a *controlled invariant subspace* for Σ_σ.

Proposition 1. *Given a family $\Sigma = \{\Sigma_i\}_{i\in I}$ of linear systems of the form (2), a subspace $\mathcal{V} \subseteq \mathcal{X}$ is a robust controlled invariant for Σ if and only if there exists a family $F = \{F_i,\ i = 1, \dots, N\}$ of feedbacks $F_i : \mathcal{X} \to \mathcal{U}$, with $i = 1, \dots, N$, such that $(A_i + B_iF_i)\mathcal{V} \subseteq \mathcal{V}$ for all $i = 1, \dots, N$. Any family F of that kind is called a* family of friends *of $\mathcal{V}$.*

If $\mathcal{K} \subseteq \mathcal{X}$ is a subspace, the set $V(A_{i\in I},\ B_{i\in I}, \mathcal{K})$ of all robust controlled invariant subspaces contained in $\mathcal{K}$ forms a semi-lattice with respect to inclusion and sum of subspaces; therefore $V(A_{i\in I},\ B_{i\in I}, \mathcal{K})$ has a maximum element, usually denoted by $\mathcal{V}^*(A_{i\in I},\ B_{i\in I}, \mathcal{K})$ or simply by $\mathcal{V}^*$ if no confusion arises. An algorithm to compute $\mathcal{V}^*(A_{i\in I},\ B_{i\in I}, \mathcal{K})$ is reported in [4] and the same was already given in [12] and proved to hold, under suitable hypothesis, also in the case of infinite families of systems.

Definition 2. Given a family $\Sigma = \{\Sigma_i\}_{i\in I}$ of linear system of the form (2), a subspace $\mathcal{S} \subseteq \mathcal{X}$ is called a *robust conditioned invariant subspace for* Σ, or a *robust* $(A_{i\in I},\ C_{i\in I})$*-invariant subspace*, if $A_i(\mathcal{S} \cap \operatorname{Ker} C_i) \subseteq \mathcal{S}$ for all $i = 1, \dots, N$. If Σ_σ is a switching linear system of form (1) defined by the elements of Σ, any robust conditioned invariant subspace $\mathcal{S}$ for Σ is said to be a *conditioned invariant subspace* for Σ_σ.

If $\mathcal{M} \subseteq \mathcal{X}$ is a subspace, the set $S(A_{i\in I},\ C_{i\in I}, \mathcal{M})$ of all robust conditioned invariant subspaces containing $\mathcal{M}$ forms a semi-lattice with respect to inclusion and intersection of subspaces; therefore $S(A_{i\in I},\ C_{i\in I}, \mathcal{M})$ has a minimum element, usually denoted by $\mathcal{S}^*(A_{i\in I},\ C_{i\in I}, \mathcal{M})$ or simply by $\mathcal{S}^*$ if no confusion arises. An algorithm to compute $\mathcal{S}^*(A_i, C_i, \mathcal{M})$ can easily be obtained by duality from the corresponding algorithm for $\mathcal{V}^*(A_i, B_i, \mathcal{K})$.

Notations. In the rest of the paper, given a family $\Sigma = \{\Sigma_i\}_{i\in I}$ of linear systems of the form (2), we will denote, respectively, by $\mathcal{K}$ the subspace $\mathcal{K} = \bigcap_{1,\dots,N} \operatorname{Ker} C_i$ and by $\mathcal{B}$ the subspace $\mathcal{B} = \sum_{1,\dots,N} \operatorname{Im} B_i$. Accordingly, $\mathcal{V}^*$ and $\mathcal{S}^*$ will usually be understood, if no confusion arises, to denote the maximum robust controlled invariant subspace for Σ contained in $\mathcal{K}$ and, respectively, the minimum conditionally invariant subspace for Σ containing $\mathcal{B}$.

Given a subspace $\mathcal{M} \subseteq \mathcal{V}^*$, we can consider the set $V(\mathcal{M})$ of all the robust controlled invariant subspaces containing $\mathcal{M}$ and contained in $\mathcal{K}$. A key property of $V(\mathcal{M})$, when $(\mathcal{V}^* \bigcap \mathcal{S}^*) \subseteq \mathcal{M}$ holds, is given below.

Proposition 2. *Given a family $\Sigma = \{\Sigma_i\}_{i\in I}$ of linear systems of the form (2), with the same notation as above, let $\mathcal{M} \subseteq \mathcal{X}$ be a subspace such that the following condition holds.*

$$\left(\mathcal{V}^* \bigcap \mathcal{S}^*\right) \subseteq \mathcal{M} \subseteq \mathcal{V}^*. \tag{3}$$

Then, the set $V(\mathcal{M})$ of all robust controlled invariant subspaces for Σ containing $\mathcal{M}$ and contained in $\mathcal{K}$ is a lattice with respect to inclusion, sum and intersection of subspaces. As a consequence, $V(\mathcal{M})$ has a minimum element.

Proof. It is enough to show that the intersection of two elements of $V(\mathcal{M})$ is a robust controlled invariant subspace and this follows from [13, Theorem 2.2] using (3).

In the hypothesis of Proposition 2, we denote by $\mathcal{V}_*(\mathcal{M})$ the minimum element of the set $V(\mathcal{M})$. It is known that $\mathcal{V}_*(\mathcal{M}) = \mathcal{V}^* \bigcap \mathcal{S}^*(A_{i\in I}, C_{i\in I}, \mathcal{M} + \mathcal{B})$.

Proposition 3. *Given a family $\Sigma = \{\Sigma_i\}_{i\in I}$ of linear systems of the form (2), let $\mathcal{M} \subseteq \mathcal{X}$ be a subspace such that condition (3) holds and let $\mathcal{V} \subseteq \mathcal{V}^*$ be a robust controlled invariant subspace for Σ with $\mathcal{M} \subseteq \mathcal{V}$. Then, any family F of friends of $\mathcal{V}$ is a family of friend also of $\mathcal{V}_*(\mathcal{M})$.*