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Nonovershooting and nonundershooting exact output regulation ☆

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ABSTRACT

We consider the classic problem of exact output regulation for a linear time invariant plant. Under the assumption that either a state feedback or measurement feedback output regulator exists, we give design methods to obtain a regulator that avoids overshoot and undershoot in the transient response.

1. Introduction

The problem of output regulation is central to modern control theory. The basic problem considers a multivariable linear time invariant (LTI) plant that is subject to known external disturbances, and which is desired to track a known reference signal. The reference signals and external disturbances are modelled by two independent exosystems. The aim of the problem is to design a feedback controller which internally stabilises the plant while rejecting the disturbances and ensuring the output converges asymptotically to the desired reference signal. The problem has a long history, and extensive compilations of results are given in [1,2].

A special case of the output regulation problem is that of designing a control law to ensure the plant output takes a known constant reference value, and also exhibits a desirable transient response, in particular the absence of overshoot or undershoot. Much of the literature for this problem has concerned single-input single-output systems (SISO). Darbha and Bhattacharyya [3] showed how to design a two parameter feedback controller for an LTI continuous-time plant that renders the step response nonovershooting. Bement and Jayasuriya [4] gave an eigenvalue assignment method to obtain a nonovershooting LTI state feedback controller for continuous-time plants with one nonminimum phase zero. In [5] conditions are given for the existence of a controller to achieve a sign invariant impulse response, and hence also a nonovershooting step response. However such an approach is inherently conservative, because a sign invariant impulse response (and hence also a monotonic step response) is not necessary to avoid undershoot or overshoot.

To date there have been few papers offering analysis or design methods for undershoot or overshoot in the step response of multi-input multi-output (MIMO) systems. A recent contribution in this area is [6], which gave conditions under which a state feedback controller could be obtained to yield a nonovershooting step response for LTI MIMO systems; this design method is applicable to nonminimum phase systems, and could be applied to both continuous-time and discrete-time systems. In [7] it was shown that the state feedback law can be implemented in conjunction with a dynamic observer; the nonovershooting property was seen to be preserved if the initial observer error is sufficiently small. In [8] the design method of [6] was modified to achieve a step response for MIMO systems that is both nonovershooting and nonundershooting.

For the general problem of exact output regulation, there have been only a few papers offering design methods to deliver a desirable transient response. Saberi et al. [9] gave a general framework for optimising transient performance in regulation problems. By defining a performance index involving the energy of the error signal, they introduced several optimal and suboptimal control problems to find control laws that achieve output regulation and also obtain the infimum of this performance index. The authors noted that in some problems it was necessary to employ high-gain

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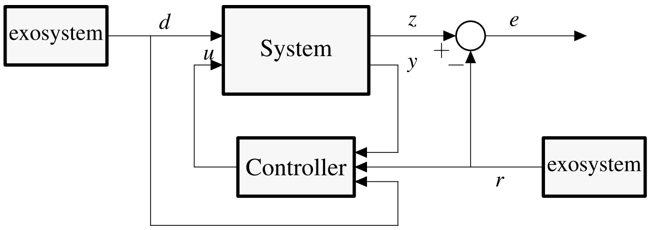

Fig. 1. Output feedback control architecture.

feedback controllers. More recently, Zhang and Lan [10] considered output regulation for SISO systems and employed the composite nonlinear feedback (CNF) technique of [11,12] to obtain a nonlinear state feedback control law that offered improved transient response, relative to that achievable with a linear control law. We note however that neither the methods of [9] nor [10] were able to avoid overshoot or undershoot in the transient response of the tracking signal.

In this paper we seek to adapt the multivariable design methods of [6,8] to the general problem of exact output regulation. We assume that the problem of output regulation by state feedback is solvable, i.e. there exists a linear state feedback controller that internally stabilises the plant and achieves output regulation. In this case we show that if there exists a state feedback controller that yields a nonovershooting response for a step reference, then a state feedback output regulator can be obtained to deliver a nonovershooting output regulation. Secondly, for the problem of output regulation by measurement feedback, we show that if the problem is solvable and if there exists a dynamic observer that yields a nonovershooting response for a constant step reference, then a measurement feedback output regulator can be found to deliver nonovershooting output regulation. To the best of the authors' knowledge, this is the first design method that achieves multivariable exact output regulation with a nonovershooting (or nonundershooting) transient response.

Notation. Throughout this paper, the symbol 0_n represents the zero vector of length n, and I_n is the n-dimensional identity matrix. For a square matrix A, we use $\sigma(A)$ to denote its spectrum. We say that a square matrix A is *Hurwitz-stable* if $\sigma(A)$ lies within the open left-hand complex plane, and it is *anti-Hurwitz-stable* if $\sigma(A)$ lies within the open right-hand complex plane. For any real or complex scalar λ and vector v, we say that (λ, v) form an *eigenpair* of a square matrix A if $Av = \lambda v$. For any matrix A with $2n$ rows, we define $\overline{\pi}\{A\}$ and $\underline{\pi}\{A\}$ by taking the upper n and lower n rows of A, respectively. If α is a vector of length n, we use $\mathrm{diag}(\alpha)$ to denote the $n \times n$ diagonal matrix whose leading diagonal contains the entries of α.

2. Problem formulation

We consider a linear multivariable plant ruled by the equation

$$\Sigma : \begin{cases} \dot{x}(t) = A\,x(t) + B\,u(t) + H\,d(t) \\ y(t) = C_y\,x(t) + D_y\,u(t) + G_y\,d(t) \\ z(t) = C\,x(t) + D\,u(t) \end{cases} \tag{1}$$

where, for all $t \geq 0$, the signal $x(t) \in \mathbb{R}^n$ represents the state, $u(t) \in \mathbb{R}^m$ represents the control input, $y(t) \in \mathbb{R}^p$ represents the measured output, $z(t) \in \mathbb{R}^q$ represents the controlled output, $r(t) \in \mathbb{R}^\rho$ represents a reference signal and $d(t) \in \mathbb{R}^\delta$ represents a disturbance signal, as shown in Fig. 1. All the matrices appearing in (1) are appropriate dimensional constant matrices.

The disturbance input d and the reference input r are generated by two autonomous exosystems ruled respectively by

$$\Sigma_{exo,1} : \begin{cases} \dot{\eta}(t) = S_1\,\eta(t), \quad \eta(0) = \eta_0 \\ d(t) = L_1\,\eta(t) \end{cases} \quad \text{and}$$

$$\Sigma_{exo,2} : \begin{cases} \dot{\zeta}(t) = S_2\,\zeta(t), \quad \zeta(0) = \zeta_0 \\ r(t) = L_2\,\zeta(t) \end{cases}$$

where, for all $t \geq 0$, $\eta(t) \in \mathbb{R}^{n_1}$ and $\zeta(t) \in \mathbb{R}^{n_2}$, and S_1, S_2, L_1, L_2 are also appropriate dimensional constant matrices. We assume that all the eigenvalues of S_1 and S_2 are anti-Hurwitz-stable, i.e., they all have non-negative real part. This assumption does not cause any loss of generality, see [2, p. 18]; indeed, if the closed-loop system (excluding the exosystems) is internally stable, the vanishing modes of the exosystem do not affect the regulation of the output. We also assume that the states of the exosystems η and ζ are measurable, i.e., they are available to be used to generate a feedforward action in the control law.

We design a controller with measurement signal y which generates the control input signal u. Our design objective is for the reference signal r to be asymptotically tracked by the output z of the system, while minimising or eliminating the effect of the disturbance. As such, by defining the error signal

$$e(t) \stackrel{\text{def}}{=} z(t) - r(t),$$

our objective is to achieve $\lim_{t\to\infty} e(t) = 0$. We then consider a new system Σ_e obtained from Σ by considering the new output e instead of z:

$$\Sigma_e : \begin{cases} \dot{x}(t) = A\,x(t) + B\,u(t) + H\,d(t), \quad x(0) = x_0 \\ y(t) = C_y\,x(t) + D_y\,u(t) + G_y\,d(t) \\ e(t) = C\,x(t) + D\,u(t) - r(t). \end{cases}$$

It is convenient to incorporate the two exosystems into a single exosystem whose state w is defined as

$$w(t) \stackrel{\text{def}}{=} \begin{bmatrix} \eta(t) \\ \zeta(t) \end{bmatrix}$$

so that

$$\Sigma_{exo} : \begin{cases} \dot{w}(t) = S\,w(t), \quad w(0) = w_0 \\ \begin{bmatrix} d(t) \\ r(t) \end{bmatrix} = \begin{bmatrix} L_1 & 0 \\ 0 & L_2 \end{bmatrix} w(t) \end{cases}$$

where $S = \begin{bmatrix} S_1 & 0 \\ 0 & S_2 \end{bmatrix}$. By defining

$$E_w \stackrel{\text{def}}{=} \begin{bmatrix} H\,L_1 & 0 \end{bmatrix}$$

$$D_{yw} \stackrel{\text{def}}{=} \begin{bmatrix} G_y\,L_1 & 0 \end{bmatrix}$$

$$D_{ew} \stackrel{\text{def}}{=} \begin{bmatrix} 0 & -L_2 \end{bmatrix}$$

we can re-write Σ_e as

$$\Sigma_e : \begin{cases} \dot{x}(t) = A\,x(t) + B\,u(t) + E_w\,w(t), & x(0) = x_0 \\ \dot{w}(t) = S\,w(t), & w(0) = w_0 \\ y(t) = C_y\,x(t) + D_y\,u(t) + D_{yw}\,w(t) \\ e(t) = C_e\,x(t) + D_{eu}\,u(t) + D_{ew}\,w(t). \end{cases} \tag{2}$$

In order to avoid issues of well-posedness of output dynamic architectures, and in order to simplify the derivations of the tracking control law, we assume $D_y = 0$. This assumption does not lead to a significant loss of generality, as shown in [2, p. 16]. For design purposes we will also consider the *nominal plant* Σ_{nom} which arises when both exosystems are excluded from consideration. In this case Σ_e simplifies to the homogeneous system

$$\Sigma_{nom} : \begin{cases} \dot{\tilde{x}}(t) = A\,\tilde{x}(t) + B\,\tilde{u}(t), \quad \tilde{x}(0) = \tilde{x}_0 \\ \tilde{e}(t) = C_e\,\tilde{x}(t) + D_{eu}\,\tilde{u}(t). \end{cases} \tag{3}$$

For this system, the problem of exact output regulation consists of driving the system state to the origin from some arbitrary nonzero initial condition. In later sections we will consider control methodologies that regulate the nominal plant with desirable transient performance, and then consider the conditions under which these control methods can also be used to achieve the same desirable transient performance when applied to Σ_e. Next