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Mode-observability degree in discrete-time switching linear systems

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ABSTRACT

In this paper, we consider the problem of identifying the active mode of a switching linear system from data sequences of a finite length. The results combine elements from canonical correlation analysis and subspace projection methods. In addition to providing insight into the geometric meaning of the problem, the results turn out to be of practical relevance whenever mode-identification is addressed in the presence of noisy-corrupted measurements. In particular, the results can be used to provide conditions under which mode-identification occurs independently of a prescribed noise level.

1. Introduction

Recently, progress in both research and industrial areas has resulted in the design and analysis of increasingly complex dynamic systems. In this context, considerable attention has been devoted to multi-mode, switching, and hybrid systems as a valuable modeling tool. Many physical systems can in fact be represented by switching or interpolating between locally valid models. Examples are found in several diversified fields such as servo positioning systems, mechanical engines with gear transmission, and communication protocols [1–3]. Switching between multiple models also arises naturally and plays a fundamental role when faulty mode dynamics are introduced to model systems subject to failures [4,5]. Similar ideas can finally be found in problems of state estimation and observer design for multimodal systems and/or under switching measurement equations [6,7].

Among the properties relevant to the analysis of multi-mode and switching systems, *mode-observability* plays a central role. Mode-observability essentially refers to the problem of identifying (reconstructing) the active mode of the system from available output measurements [8]. It is therefore a central question when the reconstruction of the mode itself is requested (e.g. in a fault detection setting). In addition, it also plays a central role whenever the estimate of the current mode in a switching system is used for the activation of an appropriate controller or state-observer.

Several results have been reported on mode-observability for the discrete time case along with centralized (see, e.g., [8–12]) or distributed (see [13]) mode-identification algorithms. The continuous time setting has been faced in [14,15] in the more general framework of invertibility of a switching system (wherein the active mode has to be reconstructed together with the system continuous input). These results, however, only provide a *yes-or-no* answer to the mode-observability question, giving no information about the mode-observability *degree* of a given system. Only recently, in the context of system invertibility, the robustness with respect to initial conditions and the possible presence of disturbances have been studied for continuous-time systems (see [16]).

Information about the *distinguishability degree* among different system modes is crucial whenever mode-identification is addressed in the presence of measurement noises, in order to quantify how noisy-corrupted measurements may affect mode-identification in practice. In fact, even in the case when two modes can be always distinguished in the absence of noises, the latter can prevent from distinguishing two modes if they are *too similar* and/or if the continuous state of the system is *too small*. A synthetic characterization of such properties is the subject of this paper.

In the general case when some modes of the system are not distinguishable for all the continuous states, several insights into the geometry of the problem are provided that characterize the continuous state domain in terms of distance from the *indistinguishability set*. Such results could be exploited in the control design stage. In fact, the possibility of quantitatively measuring the degree of mode-observability of a given switching system is a fundamental ingredient for characterizing the performance achievable by

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control and estimation algorithms which rely on mode identification [7,17–19].

As in classical state-observability problems, where some components of the state may be more difficult to observe than others, one may find systems where some of the modes are easy to observe whereas some others are not. In this respect, a key feature of the proposed approach is that it allows one to quantify the observability degree of each mode of the system with respect to any other mode. This is of special interest, for example, in fault detection, when one can give different importance to a false alarm (estimated faulty mode when the system is in the nominal configuration) with respect to a missed detection (estimated nominal mode when, instead, a fault has occurred) [4].

A preliminary step in this direction was taken in [20]. The results there proposed, however, are mainly oriented towards switching nonlinear systems and, as such, do not take full advantage of linearity. In this paper, we introduce a mode-observability index which is specific for switching linear systems, and involves elements from canonical correlation analysis and subspace projection methods.

The remainder of the paper is as follows. In Section 2, we describe the framework under consideration and give a precise characterization of distinguishability. After providing some background material (Section 3), some results about the degree of distinguishability are presented in Section 4. In Section 5, such results are related to least-squares mode estimation in a noisy environment. An example is finally given in Section 6 to substantiate the analysis. All the proofs are given in the Appendix.

2. Mode-observability

Consider the following switching linear system

$$x_{t+1} = A_{\rho_t} x_t, \quad y_t = C_{\rho_t} x_t \tag{1}$$

where $t \in \mathbb{Z}_+ = \{0, 1, \ldots\}$, $x \in R^n$ is the state, $y \in R^m$ is the output and $\rho : \mathbb{Z}_+ \mapsto \mathcal{Q}$ is the switching signal, i.e. the signal which identifies the index of the active subsystem at each time instant. $\mathcal{Q}$ is a finite index set, whose elements are referred to as the *modes*.

To begin with, a notion of *distinguishability* is introduced. Distinguishability refers to the question of whether different process modes may give rise to the same output trajectory. We make this concept precise. Let N be a given positive integer, and let

$$\mathcal{O}_q \triangleq \begin{pmatrix} C_q \\ C_q A_q \\ \vdots \\ C_q A_q^{N-1} \end{pmatrix} \tag{2}$$

be the Nth order observability matrix associated with q. The following definition is in order.

Definition 1. Mode q is said to be indistinguishable from mode p in N steps on $x \in \mathbb{R}^n$ if $\exists v \in \mathbb{R}^n$ such that $\mathcal{O}_p v = \mathcal{O}_q x$; otherwise it is said to be distinguishable.

Let now

$$\Xi_{qp} \triangleq \left\{ x \in \mathbb{R}^n | \exists v \in \mathbb{R}^n \text{ such that } \mathcal{O}_p v = \mathcal{O}_q x \right\} \tag{3}$$

be the set of states on which mode q is indistinguishable from mode p. Such a set can be obtained by projecting onto the first n components the set $\ker(\mathcal{O}_q \quad -\mathcal{O}_p)$, where, given a matrix A, $\ker A$ denotes the null space of A. The latter consideration highlights the fact that distinguishability is not a symmetric relation. In other words, one may have situations in which q is distinguishable from p but not vice versa, i.e., in general Ξ_{pq} need not be equal to Ξ_{qp}. Notice also that, because of linearity, the vector 0 always belongs to Ξ_{qp}.

The question of distinguishability is intimately connected to the so-called (initial) *mode-observability* problem, which is the problem of identifying the active mode of a switching system assuming that the process mode stays constant during the observation interval [10]. To clarify the matter, let

$$y|_{t+N-1}^{t} \triangleq \operatorname{col}(y_t, y_{t+1}, \ldots, y_{t+N-1}) \tag{4}$$

be the vector obtained by sliding a window of length N along the sequence $\{y_t\}$. Thus

$$\mathcal{O}_q x_t = y|_{t+N-1}^{t} \tag{5}$$

provided that $\rho_k = q$ for $k = t, t+1, \ldots, t+N-1$. Hence, distinguishability of q from p on the state x_t is the same as saying that one can rule out p when observing $y|_{t+N-1}^{t}$. Mode-identification can then be obtained by determining which of the candidate modes is consistent with the observation vector. This can be achieved simply by letting

$$\mathcal{Q}_t \triangleq \left\{ p \in \mathcal{Q} : \operatorname{rank} \mathcal{O}_p = \operatorname{rank} \left(\mathcal{O}_p \quad y|_{t+N-1}^{t} \right) \right\}. \tag{6}$$

Under distinguishability, i.e., when x_t does not belong to Ξ_{qp} for any $p \neq q$, $\mathcal{Q}_t$ returns a singleton which is the active mode of the system [8,11].

Unfortunately, the analysis above does not give conclusive information when, as always happens in practice, noise is present in the measurements. Let

$$z_t = y_t + \eta_t \tag{7}$$

be the actual measured signal, where $\eta \in R^m$ denotes the measurement noise. The effect of η on mode identification is that $z|_{t+N-1}^{t}$ need not belong to any of the subspaces $\operatorname{im} \mathcal{O}_p$, where, given a matrix A, $\operatorname{im} A$ denotes its range. Accordingly, when (6) is applied to z instead of y, $\mathcal{Q}_t$ may return the empty set. This essentially suggests that, from a practical viewpoint, it is not sufficient to determine whether or not distinguishability holds. On the opposite, one needs to obtain some quantitative measure for distinguishability and analyze how this affects mode-identification in the presence of noise.

The remainder of this paper is devoted to developing a systematic answer to these issues. In this respect, the basic idea is as follows. Consider a mode q, and let

$$\pi_p(\mathcal{O}_q x) \triangleq \min_{v \in \mathbb{R}^n} \|\mathcal{O}_p v - \mathcal{O}_q x\| \tag{8}$$

where $\|\cdot\|$ denotes Euclidean norm. Notice that the action of minimizing the term on the right hand side of (8) is essentially that of determining how close p may appear to q when observing $\mathcal{O}_q x$. Accordingly, (8) can be taken as a representative measure of the *degree* of distinguishability of q with respect to p at x. Notice, for instance, that $\pi_p(\mathcal{O}_q x)$ takes value 0 if and only if $x \in \Xi_{qp}$. The intuition for (8) is that the larger the values taken on by $\pi_p(\mathcal{O}_q x)$, the more negligible the effect of the measurement noise will be when the active mode of the system is q.

This is indeed the case as we show in the next sections. More fundamentally, in the analysis to follow we show that the proposed measure for distinguishability can be expressed in the form of a bound which depends on the Euclidean distance of x to the set Ξ_{qp} where the degree of distinguishability of q with respect to p is minimum. We will see that this makes it possible to determine, for each mode of the system, the regions of the space where mode-identification occurs *independently* of a prescribed noise level.

3. Background

We begin by providing some technical results necessary for the developments of Section 4. This background material is essentially