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Stabilization and synchronization for a heterogeneous multi-agent system via harmonic control

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ABSTRACT

In this paper, a class of heterogeneous multi-agent systems with directed cycle graphs is investigated. The stabilization and synchronization problems via harmonic control are solved under some conditions and the feedback control gains are constructively designed. Several numerical examples are presented to demonstrate the effectiveness of the proposed method.

1. Introduction

Multi-agent systems have been widely investigated in the past decade, whose main properties lie in the collective behavior and decentralized control based on neighborhood information. The models of multi-agent systems include the Vicsek model [1,2], Kuramoto model [3,4] and linear dynamical networks [5–9]. Actually, multi-agent systems can be regarded as a kind of complex systems or the extension of the classical large-scale systems [10–12]. But multi-agent systems focus on structures of networks and local information feedback including neighbor-state-feedback and self-state-feedback. Designing decentralized control protocol with local information to realize stability or synchronization is a basic problem in complex systems [13,14]. Actually, designing the gains of neighbor-state-feedback is just regulating interconnection gains, which has been applied in the plague control of some power networks [15,16]. An interesting theoretical question is whether the whole network can be stabilized only by tuning interconnection gains. For interconnected continuous systems composed of two linear subsystems, Duan et al. [17,18] solved the problem completely, in which designing interconnection gains is called harmonic control. For discrete-time systems, similar results were obtained in [19]. However, how to design interconnection gains to stabilize a complex dynamic networks composed of more than two subsystems is still an open problem.

Synchronization is another important issue of complex systems (see [20–24] and reference therein), which is closely related to the consensus problem of multi-agent systems [25]. It is worth noting that, in the above literature, the subsystems have the same dynamic model. But in practice, many multi-agent systems have different subsystems, which are called heterogeneous multi-agent systems in recent years [26–29]. For example, agents of flocks or satellite clusters might have different dynamic equations from each other due to their different masses or different structures. Moreover, in the references on multi-agent systems, it is often assumed that each agent can take self-state-feedback, that is, each subsystem can use its own state information in the local feedback control. How to use the interconnected feedback instead of self-state-feedback to realize consensus or synchronization is also an important research issue.

In this paper, we investigate the stabilization and synchronization via harmonic control for a class of heterogeneous multi-agent systems composed of subsystems with the general linear dynamics. Here, we only consider directed cycle graphs, which can be found in some biochemical dynamical networks [22,30]. Sufficient conditions are obtained for stabilization and synchronization via harmonic control. These results show that stability and synchronization can be realized by designing interconnection gains instead of decentralized self-state-feedback gains. Several numerical examples are presented to show the effectiveness of the proposed method.

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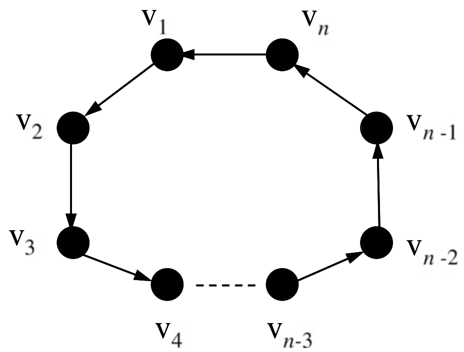

Fig. 1. The directed cycle graph.

2. Statement of problems

Multi-agent systems are composed of many subsystems connected by networks, through which state or output information is transmitted among the interconnected subsystems. Consider the network with the direct cycle graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ shown in Fig. 1, where $\mathcal{V} = \{v_1, v_2, \dots, v_n\}$ is a set of nodes and $\mathcal{E} \subset \mathcal{V} \times \mathcal{V}$ a set of edges. An edge of $\mathcal{G}$ is denoted by $e_{ij} = (v_i, v_j)$, which means node v_j can receive information from v_i.

Suppose each node of the graph is a subsystem with different dynamics

$$\dot{x}_i = A_i x_i + B_i u_i \quad (i = 1, 2, \dots, m), \tag{1}$$

where $x_i \in R^{n_i}$, $u_i \in R^{p_i}$, $A_i \in R^{n_i \times n_i}$, $B_i \in R^{n_i \times p_i}$. This kind of dynamical networks is called heterogeneous multi-agent systems.

Two problems are of concern in this paper. One is the stabilization and the other synchronization.

The problem of *stabilization via harmonic control* is to design decentralized controllers

$$u_1 = K_1 x_m, \qquad u_i = K_i x_{i-1} \quad (i = 2, 3, \dots, m) \tag{2}$$

such that the closed-loop system is globally asymptotically stable, i.e. $x_j(t) \to 0$ as $t \to +\infty$ for all initial values and $j = 1, 2, \dots, m$.

For the synchronization problem, we assume $n_i = n$ $(i = 1, 2, \dots, m)$. The problem of *synchronization via harmonic control* is to design decentralized controller (2) such that all the subsystems are in agreement asymptotically, i.e. $x_i(t) - x_j(t) \to 0$ as $t \to +\infty$ for all initial values and $i, j = 1, 2, \dots, m$.

Remark 1. For the stabilization via harmonic control defined above, some design methods have been given for the case of $m = 2$ [17–19,31]. But for the general case with $m > 2$, the problem is still open. Moreover, the synchronization via harmonic control has not been addressed for multi-agent systems.

3. Main results

Consider the heterogeneous multi-agent system composed of single-input controllable subsystems. Assume that each subsystem is modeled by

$$\dot{x}_i = A_i x_i + b_i u_i, \tag{3}$$

for $i = 1, 2, \dots, m$, where $x_i \in R^{n_i}$, $u_i \in R$, $A_i \in R^{n_i \times n_i}$, $b_i \in R^{n_i}$ and $(A_i,\ b_i)$ is controllable.

Theorem 1. *Consider the heterogeneous multi-agent system composed of single-input controllable subsystems (3) with the directed graph shown in Fig. 1. Write the characteristic polynomial of each subsystem as* $\det(sI_{n_i} - A_i) = \sum_{j=0}^{n_i} c_{ij} s^{n_i - j}$, *where* $c_{i0} = 1$ $(i = 1, 2, \dots, m)$. *Let* l_i *be any integer satisfying* $0 \le l_i \le n_i$ *for* $i = 1, 2, \dots, m$ *and* $\tilde{n} = \min_{1 \le i \le m}\{n_i\}$. *If polynomial*

$$f(s) := \sum_{l_1 l_2 \cdots l_m = 0} c_{1 l_1} c_{2 l_2} \cdots c_{m l_m} s^{n_1 + n_2 + \cdots + n_m - \tilde{n} - (l_1 + l_2 + \cdots + l_m)} \tag{4}$$

is Hurwitz, then the problem of stabilization via harmonic control is solvable.

Remark 2. Even if every subsystem is unstable, it is possible that (4) is Hurwitz. For example, let the characteristic polynomials of three subsystems be $s^4 + 8s^3 + 9s + 2s + 5$, $s^3 + 3s^2 - s + 9$ and $s^2 - s + 9$. It is easy to check that each subsystem is unstable, but

$$\begin{aligned} f(s) &= \sum_{l_1 l_2 l_3 = 0} c_{1 l_1} c_{2 l_2} c_{3 l_3} s^{7 - (l_1 + l_2 + l_3)} \\ &= s^7 + 10\, s^6 + 30\, s^5 + 121\, s^4 \\ &\quad + 135\, s^3 + 188\, s^2 + 58\, s + 45 \end{aligned} \tag{5}$$

is Hurwitz stable, where $c_{i0} = 1$, $1 \le l_i \le n_i$ and $i = 1, 2, 3$. For the special case of $m = 2$, the condition of Theorem 1 is reduced to that polynomial

$$\begin{aligned} f(s) &= \sum_{l_1 l_2 = 0} c_{1 l_1} c_{2 l_2} s^{n - (l_1 + l_2)} \\ &= s^n + (c_{11} + c_{21}) s^{n-1} + (c_{12} + c_{22}) s^{n-2} \\ &\quad + \cdots + (c_{1n} + c_{2n}) \end{aligned} \tag{6}$$

is Hurwitz, where $n = \max\{n_1,\ n_2\}$ and $c_{ij} = 0$ if $j > n_i$. Thus it follows that $c_{11} + c_{21} < 0$, which is exactly the necessary and sufficient condition $\mathrm{trace}(A_1) + \mathrm{trace}(A_2) < 0$ for stabilization via harmonic control obtained in [17]. Although for $m = 2$ our condition is stronger than that of [17], for $m > 2$ we have not found any report on the stabilization using interconnection gains. Theorem 1 gives a sufficient condition of the stabilizability via harmonic control for multi-agent systems for the first time. But it will be more interesting and challenging to explore necessary and sufficient conditions. Moreover, if the interconnected graph is a general graph or a graph with a switching topology, the stabilization problem with harmonic control will be more complex. These problems are wealth investigating in the future work.

Before the proof of Theorem 1, the following lemmas are needed.

Lemma 1. *Let* l_i *be any integer satisfying* $0 \le l_i \le n_i$ *for* $i = 1, 2, \dots, m$. *Let*

$$U = \{(l_1, l_2, \dots, l_m) | l_1 l_2 \cdots l_m = 0\},$$
$$U_1 = \{(l_1, l_2, \dots,\ l_{m-1}, l_m) | l_1 l_2 \cdots l_{m-1} = 0\}$$

and

$$U_2 = \{(l_1, l_2, \dots, l_{m-1},\ l_m) | l_m = 0,\ 1 \le l_i \le n_i,\ i = 1, 2, \dots, m-1\}.$$

Then $U = U_1 \cup U_2$ *and* $U_1 \cap U_2 = \emptyset$.

The proof is straightforward and omitted here.

Lemma 2. *Let* l_i *be any integer satisfying* $0 \le l_i \le n_i$ *for* $i = 1, 2, \dots, m$ *and* $\tilde{n} = \min_{1 \le i \le m}\{n_i\}$. *Set* $c_{i0} = 1$ $(i = 1, 2, \dots, m)$ *and Eq. (7) is given in Box I. Then*

$$\det \Gamma_m(s) = \sum_{l_1 l_2 \cdots l_m = 0} c_{1 l_1} c_{2 l_2} \cdots c_{m l_m} \times s^{n_1 + n_2 + \cdots + n_m - \tilde{n} - (l_1 + l_2 + \cdots + l_m)}. \tag{8}$$