Systems & Control Letters 66 (2014) 51–57

Contents lists available at ScienceDirect

Systems & Control Letters

journal homepage: www.elsevier.com/locate/sysconle

Absolute exponential stability and stabilization of switched nonlinear systems

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ARTICLE INFO

Article history:
Received 31 December 2012
Received in revised form
6 September 2013
Accepted 2 December 2013
Available online 17 February 2014

Keywords:
Absolute exponential stability
Switched nonlinear systems
Multiple Lyapunov functions
Average dwell time switching

ABSTRACT

This paper is concerned with the problems of absolute exponential stability and stabilization for a class of switched nonlinear systems whose system matrices are Metzler. Nonlinearity of the systems is constrained in a sector field, which is bounded by two odd symmetric piecewise linear functions. Multiple Lyapunov functions are introduced to deal with the stability of such nonlinear systems. Compared with some existing results obtained by the common Lyapunov function approach in the literature, the conservatism of our results is reduced. All present conditions can be solved by linear programming. Furthermore, the absolute exponential stabilization for the considered systems is designed by the state-feedback and average dwell time switching strategy. Two examples are also given to illustrate the validity of the theoretical findings.

1. Introduction

A switched system consists of a family of dynamical subsystems and a switching rule that determines the switching among them. Such a system has attracted much attention during recent years because it arises in many engineering applications, for example, constrained robotics [1], transmission and stepper motors [2], computer disk drives [3], and automated highways [4], to list a few. Stability and stabilization are two fundamental and important research issues in the control community. Various approaches have been proposed for solving these problems of switched systems, such as the multiple Lyapunov function (MLF) approach [5–7], the common Lyapunov function (CLF) approach [8,9], the switched Lyapunov function approach [10,11], and so on [12,13].

Recently, a special class of switched nonlinear systems, whose system matrices are Metzler and nonlinear functions satisfy the sector conditions, was studied in [14–17]. The reader is referred to the literature [18–20] for more detailed information. Note that the systems considered in these papers have extensive applications in Hopfield neural networks [21], Lotka–Volterra ecosystems [22], and variable structure systems [23], to mention a few. In [14,15], two improved CLFs were proposed, respectively, to ensure the absolute stability (ABST) of switched systems with arbitrary switching signal and any admissible sector nonlinearities. In [16], a control law guaranteeing the corresponding system to be uniformly ultimately bounded was constructed based on the CLF approach. ABST of switched nonlinear systems was further discussed in [17], where the developed conditions were more simpler than those in [14].

It is well known that the MLF approach is relatively less conservative than the CLF approach. The basic idea is that MLFs, which correspond to each subsystem or certain region in the state space, are pieced together to produce a non-traditional Lyapunov function. The traditional Lyapunov function requires that its overall energy decreases to zero along the system trajectories, while the MLF only requires non-positive time derivatives along the state trajectories for a certain subsystem.

On the other hand, it should be pointed out that, when designing a control system, one is concerned not only with the stability of the system but also with the convergence rate. A fast convergence rate in the system state is usually preferred. For example, in most of applications of the neural network and the power system, the convergence speed is expected to increase in order to cut down on the neural computing time and reduce the oscillation of power frequency. Therefore, it is important to determine the exponential stability. The concept of absolute exponential stability (AEST) given in the paper extends many types of stability such as global asymptotical stability, global exponential stability, ABST, etc. Thus, AEST is a more meaningful concept.

Based on the above observations, it naturally gives rise to some questions. Can the MLF be used instead of the CLF to deal with

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http://dx.doi.org/10.1016/j.sysconle.2013.12.021

those issues? Whether AEST of the systems considered in the paper can be obtained? These topics are interesting but challenging. For switched linear systems, it is easy to obtain: (i) each subsystem is exponentially stable; (ii) an increscent ratio between Lyapunov functions of different subsystems. Thus, an extension from the CLF approach to the MLF approach is trivial, and global exponential stability or AEST is easy to obtain. However, the two aspects seem to be hard to reach for switched nonlinear systems due to nonlinearity of the systems and Lyapunov functions. These motivate us to carry out the work.

This paper studies AEST of switched nonlinear systems with sector conditions. We slightly modify the sector conditions that the upper and lower boundaries hold linear form. A formula of MLF is constructed to solve AEST of the system. The present method is also used to design the feedback law and ADT of non-autonomous systems, such that the resulting closed-loop systems are AEST. The paper extends the results in [14,17]. The remainder of the paper is organized as follows: Section 2 presents the problem formulation and preliminaries; main results are given in Section 3. In Section 4, two examples are proposed; Section 5 concludes the paper.

Notations: $\Re$ denotes the set of real numbers, $\Re^n$ represents the space of the vectors of n-tuples of real numbers, and $\Re^{n\times n}$ is the space of $n\times n$ real matrices. The interval $[0,\infty)$ in $\Re$ is denoted by $\mathbb{R}_+$. $\mathbb{N}$ and $\mathbb{N}_+$ are the sets of nonnegative and positive integers, respectively. $\|\cdot\|$ is the Euclidean norm. A^T is the transpose of matrix A. I_n is the $n\times n$ identity matrix. For v in $\Re^n$, v_i is the ith component of v, and $v \succ 0(\succeq 0)$ means that all components of v are positive(nonnegative), i.e. $v_i > 0(\geq 0)$. For A in $\Re^{n\times m}$, a_{ij} stands for the element in the ith row and the jth column of A. A matrix A is said to be a Metzler matrix if its off-diagonal elements are all nonnegative. A function $\varphi:\mathbb{R}_+\to\mathbb{R}_+$ is said to be of class $\mathcal{K}$ if it is a continuous, strictly increasing function satisfying $\varphi(0)=0$. It is of class $\mathcal{K}_\infty$ if it is unbounded.

2. Problem formulation and preliminaries

Consider the switched nonlinear system

$$\dot{x}(t) = A_{\sigma(t)} f(x(t)), \tag{1}$$

where $x=(x_1,\ldots,x_n)^T \in \Re^n$ is the system state; $f(x)=(f_1(x_1),\ldots,f_n(x_n))^T\in\Re^n$; $\sigma(t)$ is a mapping defining the switching law from $[0,\infty)$ to a finite set $S=\{1,2,\ldots,N\}$, $N\in\mathbb{N}_+$, and it is continuous from the right everywhere for a switching sequence $0\le t_0<t_1<\cdots$; $A_p\in\Re^{n\times n}$, $p\in S$. Throughout the paper, it is assumed that A_p is a Metzler matrix for each $p\in S$, unless otherwise stated. The nonlinear function $f(x)$ lies in the sector field satisfying

$$\begin{aligned}&\gamma\varsigma^2 \le f_i(\varsigma)\varsigma \le \delta\varsigma^2, \quad \forall \varsigma\in\Re,\\ &f_i(0)=0, \quad i=1,2,\ldots,n,\end{aligned} \tag{2}$$

where $0<\gamma\le 1\le\delta$. The restriction (2) was applied in [24–26] and extended the sector restriction $f_i(\varsigma)\varsigma>0$ used in [14–17]. We also relax the sector condition (2) to more general case later.

Now, we introduce some definitions and lemmas.

Definition 1. System (1) is said to be AEST if its zero solution is globally exponentially stable for any admissible nonlinear functions $f_1(x_1),\ldots,f_n(x_n)$ satisfying (2) and any switching signal.

In the paper, the switching signal is ADT switching rather than arbitrary switching in [14–17]. ADT switching can characterize a larger class of stable switching signals than arbitrary switching, and its extreme case is actually the arbitrary switching. This class of switching signal has been widely employed to solve the problems of switched systems [5–7,12,13].

Definition 2. Let $\sigma(t)$ be a switching signal and $N_\sigma(t_2,t_1)$ be the switching number of $\sigma(t)$ in time interval $[t_1,t_2]$. If there exist two constants $N_0\ge 0$ and $\tau^*>0$ such that

$$N_\sigma(t_2,t_1) \le N_0 + (t_2-t_1)/\tau^*, \tag{3}$$

then τ^* is an ADT of the switching signal $\sigma(t)$ and N_0 is the chatter bound.

Lemma 1 ([27]). *Let $A\in\Re^{n\times n}$ be a Metzler matrix. Then the following statements are equivalent:*

(i) *A is a Hurwitz matrix;*
(ii) *there exists a vector $v\succ 0$ in $\Re^n$ such that $Av\prec 0$.*

By Lemma 1, it holds that there exists a vector $v\succ 0$ in $\Re^n$ such that $Av\prec 0$ is equivalent to that there exists a vector $v'\succ 0$ in $\Re^n$ such that $A^Tv'\prec 0$.

3. Main results

This section contains three subsections. The first subsection gives AEST analysis of system (1). The second subsection presents some extensional results. The third subsection addresses stabilization design of a new defined system.

A CLF $V(x)=\Sigma_{i=1}^n \lambda_i\int_0^{x_i} f_i^\iota(\varsigma)d\varsigma$ was employed in [14], where $\iota>0$ is a given rational number with odd numerator and denominator, and $\lambda=(\lambda_1,\ldots,\lambda_n)\succ 0$. Along the system (1), the time derivative of the CLF

$$\dot{V}(x) = \sum_{i,j=1}^{n} \lambda_i a_{ij}^{(p)} f_i^\iota(x_i) f_j(x_j) \tag{4}$$

is negative definite for each $p\in S$, where $a_{ij}^{(p)}$ is the i row j column of A_p. This paper presents MLF as follows:

$$V_p(x) = \sum_{i=1}^{n} \lambda_i^{(p)} \int_0^{x_i} f_i(\varsigma)d\varsigma, \tag{5}$$

where $\lambda^{(p)}=(\lambda_1^{(p)},\ldots,\lambda_n^{(p)})^T\succ 0$.

3.1. AEST analysis

In the first subsection, we give AEST analysis of system (1).

Theorem 1. *If there exist a constant $\mu>0$ and a vector $v^{(p)}=(v_1^{(p)},\ldots,v_n^{(p)})^T\succ 0$ such that*

$$A_p v^{(p)} + \mu v^{(p)} \prec 0, \tag{6}$$

holds $\forall p\in S$, then under the ADT satisfying

$$\tau \ge \tau^* = \frac{\ln\eta}{\alpha}, \tag{7}$$

system (1) is AEST for any admissible nonlinearity, where $\alpha=\frac{2\mu\underline{v}\cdot\underline{\omega}\gamma^2}{\overline{v}^2\delta\overline{\lambda}}$ and $\eta=\frac{\delta\overline{\lambda}}{\gamma\underline{\lambda}}$, where $\underline{v}=\min_{p\in S}\{v_i^{(p)},\ i=1,2,\ldots,n\}$, $\overline{v}=\max_{p\in S}\{v_i^{(p)},\ i=1,2,\ldots,n\}$, $\underline{\omega}=\min_{p\in S}\{\omega_i^{(p)},\ i=1,2,\ldots,n\}$, $\underline{\lambda}=\min_{p\in S}\{\lambda_i^{(p)},\ i=1,2,\ldots,n\}$, $\overline{\lambda}=\max_{p\in S}\{\lambda_i^{(p)},\ i=1,2,\ldots,n\}$, and $\omega_i^{(p)}$ and $\lambda_i^{(p)}$ are defined in (8) and (9), respectively.

Proof. By Lemma 1 and (6), there exists a vector $\omega^{(p)}=(\omega_1^{(p)},\ldots,\omega_n^{(p)})^T\succ 0$ such that

$$A_p^T\omega^{(p)} + \mu\omega^{(p)} \prec 0. \tag{8}$$