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A quadratic yield function with multi-involved-yield surfaces describing anisotropic behaviors of sheet metals under tension/compression

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ABSTRACT

A quadratic yield function which can describe the anisotropic behaviors of sheet metals with tension/compression symmetry and asymmetry is proposed. Five mechanical properties are adopted to determine the coefficients of each part of the yield function. For particular cases, the proposed yield function can be simplified to Mises or Hill's quadratic yield function. The anisotropic mechanical properties are expressed by defining an angle between the current normalized principal stress space and the reference direction with the assumption of orthotropic anisotropy. The accuracy of the proposed yield function in describing the anisotropy under tension and compression is demonstrated.

1. Introduction

Metal forming processes are widely used in the fields of aeroplane and automobile [1–3]. Yield function plays an important role in the field of metal forming. As a classical quadratic function, Mises yield function cannot describe the anisotropic deformation behavior of sheet materials. Hill's quadratic yield function [4] is also a classical anisotropic yield function, the parameters of which can be solved with four yield stresses or three R-values [5,6] for plane stress condition.

For the "anomalous phenomenon" of Hill48 yield function, Hill [7] proposed another yield function named Hill79 with the exponent of non-integer value which needs to be solved with numerical method. Then Hill90 [8] and Hill93 yield functions [9] were proposed, which have higher accuracy due to the higher flexibility. Barlat and Lian [10] proposed a yield function which has been widely used until now because of its simple form, convenience in use and relatively high accuracy. And then Yld91 was proposed [11], which can be used in the 3D stress space. For the anisotropic behavior of aluminum alloy sheets, more general expressions of Yld94 [12], Yld96 [13], etc. were proposed by Barlat et al. With the growing demand for the accuracy of the prediction of sheet metal forming process in modern industrial applications, some new yield functions with higher accuracy called "advanced anisotropic yield criteria" [14] have been proposed since 2000. Yld2000-2d yield function [15] is a typical example of the advanced yield functions, which overcomes some disadvantages of Yld94 [12] and Yld96 [13] yield functions. Yld2000-2d yield function has been widely used to describe

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the anisotropic behavior and calculate the forming limit of sheet metals [15–19]. With the methodology of linear transformations, Yld2004 yield function (Yld2004-18p and Yld2004-13p) were also proposed by Barlat et al. [20], which can be used for both 3D full stress and plane stress conditions. Yld2004 has high flexibility and can predict six or eight ears in cup drawing process [21]. Besides the above anisotropy, the tension/compression asymmetry of sheet metals is another important problem in the sheet metal forming field especially for the alloys with HCP (Hexagonal Closed Packed) structure [22–30]. Cazacu and Barlat [31] proposed a third-order yield function describing the tension/compression asymmetry behavior. Then Cazacu et al. [32] proposed the CPB06 yield function which can describe anisotropy under tension and compression loading paths especially for magnesium and titanium alloys. Plunkett et al. [33] proposed a series of yield functions called CPB06ex2, which can describe both tensile and compressive anisotropy of HCP crystal structure and cubic crystal structure by extending the CPB06 yield function. Khan et al. [34] proposed a yield function describing the anisotropic behavior and tension/compression asymmetry of the Ti-6Al-4V alloy well. Yoon et al. [35] proposed a yield function including the first, second and third stress invariants of the stress tensor, which can predict the SD effect and anisotropic behavior in tension and compression.

As mentioned above, in order to develop an accurate anisotropic yield function, an adequate amount of material parameters should be incorporated to ensure high flexibility. So if a single equation is adopted to represent a yield function, it might be very complicated. In order to ensure the describing accuracy of the anisotropic behavior of sheet metals, the simplicity of the form, and the convenience of parameter solution, some improvements on Hill's quadratic yield function have been performed [36–40].

Vegter and Boogaard [41] proposed an anisotropic plane stress yield function based on the second-order Bezier curves interpolation. Due to the high flexibility ensured by a large number of parameters, this yield function can be very accurate in describing the anisotropic behavior. As for the implementation into the FEM software, the CPU times of Vegter's yield function are comparable with that of the Hill's quadratic yield function [41]. Hu [40] proposed a quadratic yield function, with which the concept of multiple yielding systems was introduced. Similar to Vegter's yield function, Hu's yield function is also a piecewise function. In Hu's yield function, it is difficult to ensure the accuracy of the yield function at connected points, and sometimes the resulting yield function is non-convex [40]. Since each part has the same form of Hill's quadratic function, Hu's yield function is also point-symmetric, so the tension-compression asymmetry cannot be taken into account.

The motivation of this study is to develop a quadratic function which is easy to operate but with high accuracy. In this study, a new quadratic anisotropic yield function on the basis of Hill's quadratic yield function is proposed with the above-mentioned concept of multi-yield-surface. A piecewise function is adopted to construct the entire yield locus. Besides the common experimental material properties (such as σ_0, σ_{45}, σ_{90}, $\sigma_b$and R_0, R_{45}, R_{90}, R_b), some material properties under other loading paths are incorporated into the yield function, which ensures the high flexibility of the yield function to describe the anisotropic behavior of sheet metals accurately. The anisotropy of tension/compression asymmetry for some sheet metals can also be considered in the present yield function as well as that of tension/compression symmetry. The parameters of the proposed yield function can be obtained without numerical iteration since it is quadratic. The developed yield function has high flexibility which ensures its accuracy. Although there are certain amount of parameters to be determined, the solving process of the parameters is very easy with the help of computer as long as the needed material properties are obtained. No numerical method is needed because the function is simple (quadratic). Besides, the number of required experiments to determine the parameters of the developed yield function depends on the actual need. For the cases that there is insufficient experimental property or it is unnecessary to consider too many properties, the lacked properties can also be determined with proper assumptions. It is found that the proposed yield function has higher accuracy than some existing yield functions in describing the anisotropic properties and plastic contours of sheet metals.

2. The present yield function for plane stress condition

2.1. *The mathematical model of the yield function*

The yield locus in an arbitrary nomalized plane principal stress space is divided into 6 yield parts (named f_1, f_2, f_3, f_4, f_5 and f_6, respectively), as shown in Fig. 1, each of which will be represented by one function. For each part of the yield locus, there are three reference points to determine the corresponding function. The corresponding ranges of stress states and the reference points for each part of the yield function are listed in Table 1. In Fig. 1, σ_1 and σ_2 are the two principal stresses in the current plane principal stress space, and $\bar{\sigma}$ is the equivalent stress. The common condition of $\sigma_1 \geq \sigma_2$ is not adopted here. The new definitions of σ_1 and σ_2 are presented in Section 3 in detail.

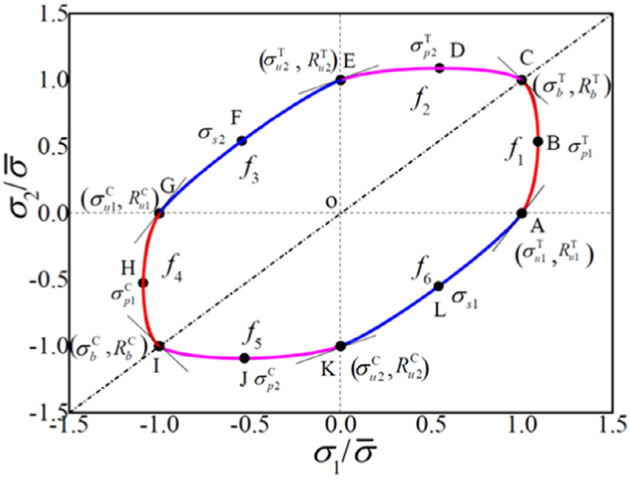

Fig. 1 – The yield surface with corresponding reference points in the normalized principal stress space.