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An Internal Expansive Stress Model of Concrete under Sulfate Attack[⋆⋆]

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ABSTRACT Concrete experiences expansive deformation during sulfate attack due to internal expansive stress. Sulfate ions enter concrete pores, react with the pore solution, and produce ettringite. The production of ettringite explains the internal expansive stress that reduces the durability of concrete. In this study, the model of internal expansive stress was achieved through the Eshelby's theory, as well as the experimental results for concrete erosion. Numerical simulation indicated that internal expansive stress is not only determined by the water-to-cement ratio of concrete and the concentration of sulfate solution, but also affected by the relaxation time of concrete.

KEY WORDS concrete, viscous, internal stress, model, erosion, micromechanics

I. Introduction

Sulfate attack is one of the major factors affecting the durability of concrete in marine environments. After entering concrete pores, sulfate ions react with the pore solution and produce calcium sulpho-aluminate—also known as delayed ettringite. The growth of delayed ettringite crystals may induce expansive stress. Such expansive stress may lead to the nucleation and growth of micro-cracks that will decrease the durability of concrete structures. Current studies on the resistance to sulfate attack focus primarily on the following three areas.

The first is the research on improving concrete permeability resistance. The primary objective of this type of research is to enhance the density of concrete so as to improve its erosion resistance. Research results[1–3] have indicated that decreasing the water-to-cement ratio of concrete (w/c) could decrease the porosity and increase the density, and accordingly improve the sulfate attack resistance of concrete. Other investigators found that adding mineral additives such as fly ash, granulated blast furnace slag, and portland-pozzolana materials during concrete mixing could significantly improve the durability of concrete[4–7].

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The second is the hydration process. Here, calcium aluminate C_3A and calcium hydroxide were controlled to reduce the production of ettringite, gypsum, etc. This decreases the internal expansive stress and reduces damage[7–10].

The third is the research into the diffusion of hazardous ions and deterioration of mechanical properties in concrete under sulfate attack. Bonakdar et al.[11] studied the diffusion of sulfate ions in the micro-hardness of blended cement materials. Sun et al.[12] further considered the effect of erosion damage on the diffusion of sulfate ions, and achieved a theoretical model with better conformity to experimental data. Their results indicated that higher concentrations of sulfate ions in erosion solution give higher diffusion velocity for the sulfate ion. Ouyang et al.[13] analyzed the evolution process of concrete surface stiffness versus time under sulfate solution erosion. Chen et al.[14] investigated sulfate erosion with ultrasonic experiments to show the correlation between cement mortar and damage as a function of time. Hekala[15] investigated the impact of magnesium ions in sulfate on the mechanical properties of concrete. They discovered that magnesium sulfate resulted in decalcification of concrete and accordingly changed the calcium silicate hydrate (C-S-H) in concrete into magnesium silicate hydrate (M-S-H). Because concrete damage is primarily caused by the expansion of ettringite, some investigators have also studied the mechanical properties of ettringite. Zhang[16] and Yang et al.[17] tested the static and dynamic moduli of delayed ettringite crystals.

Although it is known that the internal expansive stress triggers erosion damage, there is still a shortage of convincing quantitative data on the evolution of this internal expansive stress in eroding concrete. Internal expansive stress in concrete not only is determined by the concentration of the erosion solution, but also closely correlates to the pore structure and mechanical properties of the material.

In this study, it was assumed that the constitutive relation of the concrete matrix can be described by a viscoelastic approach, and the micro-damage evolution is also taken into account. Then an incremental constitutive model was provided. Meanwhile, concrete was assumed to be a two-phase composite material with voids and the concrete matrix. Based on the established incremental constitutive model, the mean field theory of mesomechanics and the Eshelby's equivalent inclusion method were utilized to create an evolution model of internal expansive stress characterized by the macro-strain of concrete. Simulation data indicated that the concentration of erosion solution, the water-to-cement ratio and the relaxation time of the material all have significant impacts on the internal expansive stress of concrete under sulfate attack.

II. The Incremental Constitutive Model of Concrete Matrix Material

Concrete can be considered as a viscoelastic material because it has a clear strain-rate effect[18]. In general, the constitutive relationship of viscoelastic materials with damage can be expressed as

$$\boldsymbol{\sigma} = [1 - D(t)] \int_0^t \boldsymbol{L}(t-\tau) : \dot{\boldsymbol{\varepsilon}}(\tau)\,\mathrm{d}\tau \tag{1}$$

where $\boldsymbol{L}$ is the fourth-order relaxation modulus tensor of matrix material and $D(t)$ is the damage degree of the material.

To study the internal expansive stress using mesomechanics, Eq.(1) was replaced by the following incremental form:

$$\Delta\boldsymbol{\sigma} = [1 - D(t+\Delta t)] \int_0^{t+\Delta t} \boldsymbol{L}(t+\Delta t-\tau) : \dot{\boldsymbol{\varepsilon}}(\tau)\mathrm{d}\tau - [1 - D(t)] \int_0^t \boldsymbol{L}(t-\tau) : \dot{\boldsymbol{\varepsilon}}(\tau)\mathrm{d}\tau \tag{2}$$

Then $\boldsymbol{L}(t+\Delta t-\tau)$ and $D(t+\Delta t)$ in Eq.(2) were expanded using the Taylor series, and the resultant second order minima were omitted. The mean value theorem was then used to obtain the following formula,

$$\begin{aligned}\Delta\boldsymbol{\sigma} = {} & [1 - D(t)] \int_0^t \left(\frac{\partial \boldsymbol{L}(t-\tau)}{\partial t} \Delta t \right) : \dot{\boldsymbol{\varepsilon}}(\tau)\mathrm{d}\tau - \frac{\partial D(t)}{\partial t} \Delta t \int_0^t \boldsymbol{L}(t-\tau) : \dot{\boldsymbol{\varepsilon}}(\tau)\mathrm{d}\tau \\ & + [1 - D(t+\Delta t)]\, \boldsymbol{L}[t+\Delta t-(t+\xi\Delta t)] : \dot{\boldsymbol{\varepsilon}}(\tau+\xi\Delta t)\,\Delta t \end{aligned} \tag{3}$$

where $0 \le \xi \le 1$.