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Nonlinear Vibration of A Loosely Supported Curved Pipe Conveying Pulsating Fluid under Principal Parametric Resonance

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ABSTRACT In this paper, the nonlinear dynamics of a curved pipe is investigated in the case of principal parametric resonance due to pulsating flow and impact with loose supports. The coupled in-plane and out-of-plane governing equations with the consideration of von Karman geometric nonlinearity are presented and discretized via the differential quadrature method (DQM). The nonlinear dynamic responses are calculated numerically to demonstrate the influence of pulsating frequency. Finally, the impact is taken into consideration. The influence of clearance on fretting-wear damage, such as normal work rate, contact ratio and impact force level, is demonstrated.

KEY WORDS curved pipes conveying fluid, DQM, parametric resonance, loose support

I. Introduction

The vibration problem of fluid-conveying pipes, both straight and curved, is an important academic field with broad industrial application, e.g., hydroelectric and nuclear power plants, suction and pressure pipes, and fuel feeding lines in aerospace, etc. As discussed in a review[1], the fluid-conveying pipe has become a paradigm in the study of stability and nonlinear systems. Thus, it is not surprising that there has been a great deal of effort devoted to investigating the mechanism of this vibration[2–11]. Variants of pipe systems have been shown to exhibit complex dynamics.

Up to now, straight pipes conveying fluid have been investigated extensively. In contrast, only a few studies have been devoted to the dynamics of curved pipes conveying fluid because the instruction of the governing equation is complicated. Refs.[1–3] have presented a perspective review of the available research on this dynamical problem. The early studies on the stability of curved pipes conveying fluid were confined to the linear aspect of this fluid-structure interaction system. With the assumption that the centerline of the curved pipe is inextensible, the dynamics and stability of curved pipes were studied extensively by Chen[1]. He claimed that in the case of clamped-clamped and pinned-pinned boundary conditions, the pipe loses stability by divergence similar to that of a straight pipe when the flow velocity or the fluid pressure exceeds a certain critical value. However, Hill and Davis[12], Doll and Mote[13] found that, if the centerline of the pipe is extensible, a clamped-clamped curved pipe does not lose stability, no matter how high the flow velocity may be. Subsequently, the inextensible, extensible and

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modified inextensible theories are presented by Misra et al.[14,15]. The dynamics of curved pipes are investigated by the finite element method (FEM). For pipes with both ends supported, the results predicted by extensible theory are remarkably close to those of the modified inextensible one. Ni and Wang[16] took up the problem of a cantilever curved pipe conveying fluid with nonlinear constraints, and the route to chaos for the pipe was obtained. In-plane and out-of-plane natural frequencies of a curved pipe conveying fluid have been analyzed by Jung and Chung[17] using the Galerkin method.

Moreover, a parametric excitation may cause the pipes to undergo dynamic instability due to parametric resonances. The dynamics of a straight pipe conveying pulsating fluid has been the subject of increasing attention. On the other hand, from a literature survey, it is found that there are significantly fewer studies on a curved pipe conveying pulsating fluid. The equations of motion of a curved pipe conveying pulsating fluid were obtained by Jung et al.[18] using the Hamilton principle. The linear stable region of the pipe was analyzed numerically by the Floquet theory. Subsequently, the out-of-plane vibration of a curved pipe due to pulsating flow was investigated experimentally by Nakamura et al.[19], and the principal parametric resonance of the out-of-plane pipe vibration was detected. The combined parametric resonance and principal parametric resonance of a curved pipe induced by pulsating flow were investigated by Ni et al.[20] with the method of multiple scales (MMS) and DQM.

After the onset of instability, the structural motions become large with proper system parameters, and hence the nonlinear effects become increasingly important, both for geometric nonlinearity and the nonlinearity induced by the impact between the pipe and the supports. The nonlinear restrained curved pipe conveying fluid was studied extensively by Wang et al.[21]. However, there are several limitations in Wang's work: (i) the effect of the motion constraints were described as a cubic-spring restraining force, meaning only the force normal to the curved pipe caused by the constraints was considered but the friction force along the curved pipe was neglected; (ii) the velocity of the internal fluid was constant (iii) the effect of damping induced by the energy dissipated in the system due to impact was not considered in the research, either.

The motivation for the current work is twofold. Firstly, by introducing internal pulsating fluid, the coupling relation between the in-plane and out-of-plane vibrations of the curved pipe conveying fluid system is to be revealed. Secondly, a more complete and widely used impact model is introduced to this loosely supported pipe system to investigate the fretting-wear damage. The governing equations of motion are derived for an extensible curved pipe conveying pulsating fluid by using the Newtonian method with the consideration of von Karman geometric nonlinearity. The governing equations show that, the out-of-plane vibrations are only coupled with the in-plane vibrations through nonlinear terms. Then, the DQM is utilized to discretize the complete pipe model, and the nonlinear dynamic equations are solved by a fourth order Runge-Kutta scheme. The parametric resonance responses are demonstrated. The loose supports may come into play in the case of parametric resonance, and the influence of support clearance on the impact response is investigated.

II. Model Development

The system under consideration is shown in Fig.1. It consists of a uniformly curved pipe of radius R, length L, external diameter d_e, internal diameter d_i, mass per unit length m, flexural rigidity EI, torsional rigidity GJ, cross-sectional area A_p and conveying fluid of mass per unit length M, flowing with a pulsating velocity U. The fluid flow is assumed to be a plug flow. The displacements are denoted as u, v, w and ϕ corresponding to the transverse in-plane displacements, transverse out-of-plane displacements, tangential displacements along the pipe and the twisting angle due to torsion, respectively. The loose supports are defined by anti-vibration bars (AVBs) positioned at the mid of the flexible pipe.

The derivation process of Misra[14] is adopted to obtain the governing equations of motion for the dynamical system.

If the extensible theory and von Karman nonlinear axial strain-displacement relation are employed, the axial force Q_z is given by

$$Q_z = \left(E + E^{\star}\frac{\partial}{\partial t}\right) A_p \left[\frac{\partial w}{\partial s} - \frac{u}{R} + \frac{1}{2}\left(\frac{\partial u}{\partial s} + \frac{w}{R}\right)^2 + \frac{1}{2}\left(\frac{\partial w}{\partial s} - \frac{u}{R}\right)^2 + \frac{1}{2}\left(\frac{\partial v}{\partial s}\right)^2\right] \tag{1}$$

where $E^{\star}$ is the coefficient of Kelvin-Voigt damping in the material.