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Dielectric Breakdown Model for an Electrically Semi-Permeable Penny-Shaped Crack in Three-Dimensional Piezoelectric Media**

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ABSTRACT The dielectric breakdown (DB) model for a penny-shaped crack under a semi-permeable boundary condition in a three-dimensional piezoelectric medium is studied. An approximate analytical solution is derived by using the boundary integral equation with extended displacement discontinuity, and the corresponding boundary element method with double iterative approaches is developed to analyze the semi-permeable crack. The effect of electric boundary conditions on crack faces is discussed on the basis of DB model. By comparing the DB model with the polarization saturation (PS) model for different piezoelectric materials, some interesting phenomena related to the electric yielding zone and local J-integral are observed.

KEY WORDS piezoelectric medium, penny-shaped crack, dielectric breakdown model, polarization saturation model, semi-permeable crack, local J-integral

I. Introduction

Piezoelectric materials can provide coupling between mechanical and electrical fields and are widely used in engineering applications. As micro-defects and cracks in piezoelectric ceramics can greatly affect their behavior and reduce their strength, extensive and intensive studies have been conducted[1–6]. It is still necessary to illustrate the internal relations between electric load and fracture behavior for piezoelectric materials. However, to extend the current fundamental fracture concepts and criteria for pure elasticity to piezoelectricity is not straightforward owing to the electro-mechanical coupling[1,2].

Built on the Dugdale model[7], the well-known polarization saturation (PS) model was proposed, which regarded a piezoelectric material as mechanically brittle and electrically ductile[8], where the electric displacement in the electric yielding zone was equal to the polarization saturation value. Fracture features predicted using the PS model were in broad agreement with the experimental observations[9]. This model was intensively studied afterwards[10–14].

In 2001, McMeeking reexamined the PS model and pointed out that the electric field strength in the PS model should correspond to the stress in the classical Dugdale model from the energy point of view[15]. Accordingly, Zhang et al.[16,17] developed the strip dielectric breakdown (DB) model to study the effect of electrical non-linearity on piezoelectric fracture. In the DB model, the electric field strength

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in the electric yielding zone equals the dielectric breakdown strength. More studies on the DB model were conducted for piezoelectric media[18–20] and electrostrictive solids[21].

Differences and similarities between the PS model and the DB model were studied and discussed according to their analytical solutions[17] and numerical solutions[19] for 2D piezoelectric media. Penny-shaped cracks in 3D piezoelectric media were investigated[20]. Results obtained under the electrically impermeable boundary condition indicated that although the PS and DB models have different physical bases, they provide similar solutions for the electric yielding zone and the local J-integral at the crack tip. The equivalence of the PS model and the DB model was concluded in terms of both the intensity factors and the local J-integral.

For the sake of simplicity, most research has focused on the electrically impermeable cracks. However, an electric field can exist everywhere, even in a crack cavity. The impermeable crack condition is only one of the several extreme cases[2]. Among these boundary conditions, the semi-permeable boundary condition is more realistic[22]. Indeed, results obtained using the PS model showed that the electric boundary condition on a crack face affects the solutions for a two-dimensional (2D) piezoelectric medium[19,23] and a three-dimensional (3D) piezoelectric medium[24].

In this paper, we report the study of a penny-shaped crack in a 3D piezoelectric medium using the DB model under a semi-permeable boundary condition theoretically and numerically. Results are compared for the electric displacement in the crack cavity, the electric yielding zone and the local J-integral obtained using the DB model and the PS model under different boundary conditions for different piezoelectric materials.

II. DB Model

A penny-shaped crack S of radius a is located on the Oxy-plane centered at origin O in a 3D transversely isotropic piezoelectric medium, as shown in Fig.1. The polarization is along the z-direction of the rectangular coordinate system. Uniformly distributed mechanical loading p_0 and electric loading ω_0 are applied on both the upper and the lower faces of crack S. It can be seen that the problem is axisymmetric owing to the symmetry of crack orientation, the material and the applied loading. Therefore, the solution is a function of coordinates r and z. We denote the crack by $S=\{0\le r\le a;\ 0\le\theta\le 2\pi\}$, and give the mechanical boundary conditions in terms of traction $p\,(r,0)$ or stress $\sigma\,(r,0)$ and the electric boundary condition in terms of the electric displacement boundary value $\omega(r,0)$ or the electric displacement $D_z(r,0)$ on the crack faces as

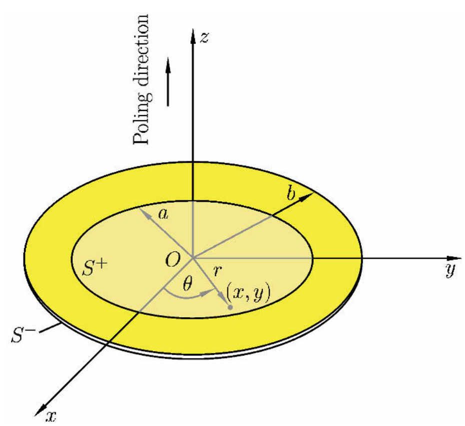

Fig. 1 Penny-shaped crack with an electric yielding zone in a transversely isotropic piezoelectric material.

$$\begin{aligned} p\,(r,0^+) &\equiv -\sigma\,(r,0^+) = p_0 \\ p\,(r,0^-) &\equiv \sigma\,(r,0^-) = -p_0 \end{aligned} \qquad (0\le r\le a) \tag{1}$$

$$\omega\,(r,0) = -D_z\left(r,0^+\right) + D^{\mathrm{c}}\,(r) = D_z\left(r,0^-\right) - D^{\mathrm{c}}\,(r) = \omega_0 \qquad (0\le r\le a) \tag{2}$$

where the superscripts '+' and '−' denote the points on the upper and lower crack faces, respectively. D^{c} is the electric displacement in the crack cavity, which is determined by the crack opening displacement (or the displacement discontinuity) and the electric potential jump (or the electric potential discontinuity) and can be expressed as[22]

$$D^{\mathrm{c}}(r) = -\kappa^{\mathrm{c}}\frac{\varphi(r,0^+)-\varphi(r,0^-)}{w(r,0^+)-w(r,0^-)} = -\kappa^{\mathrm{c}}\frac{\|\varphi(r)\|}{\|w(r)\|} \tag{3}$$

where κ^{c} is the dielectric constant of the medium inside the crack cavity, w is the displacement along the z-direction, φ is the electric potential, and $\|w(r)\|$ and $\|\varphi(r)\|$ denote the crack opening displacement and the electric potential jump across the crack faces, respectively.

The electric yielding zone is circular according to symmetry and the dielectric breakdown model[17], and is denoted as $S_{\mathrm{b}}=\{a\le r\le b;\ 0\le\theta\le 2\pi\}$, as shown in Fig.1. In this area, the electric field