Acta Mechanica Solida Sinica, Vol. 29, No. 4, August, 2016 ISSN 0894-9166
Published by AMSS Press, Wuhan, China

Isogeometric Analysis of the Nonlinear Deformation of Planar Flexible Beams with Snap-back**

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Received 10 October 2015, revision received 4 March 2016

ABSTRACT Based on the continuity of the derivatives of the Non-Uniform Rational B-Splines (NURBS) curve and the Jaumann strain measure, the present paper adopted the position coordinates of the control points as the degrees of freedom and developed a planar rotation-free Euler-Bernoulli beam element for isogeometric analysis, where the derivatives of the field variables with respect to the arc-length were expressed as the sum of the weighted sum of the position coordinates of the control points, and the NURBS basis functions were used as the weight functions. Furthermore, the concept of bending strip was used to involve the rigid connection between multiple patches. Several typical examples with geometric nonlinearities were used to demonstrate the accuracy and effectiveness of the proposed algorithm. The presented formulation fully accounts for the geometric nonlinearities and can be used to study the snap-through and snap-back phenomena of flexible beams.

KEY WORDS isogeometric analysis, NURBS, rotation-free beam, bending strip method, snap-back

I. Introduction

Flexible beams, which have a wide range of engineering applications, such as helicopter rotor blades, aircraft wings, wind-turbine blades, robot manipulators, and slender space structures[1], are a class of slender structures that can undergo large elastic deformations. With the application of flexible structures, the nonlinear phenomena occurred in the system has attracted more and more attention. Pai and Nayfeh[2] proposed new concepts of local displacements and local engineering stresses and strains, a new interpretation and manipulation of the virtual local rotations. Their theory, which was verified by using a multiple shooting method[3], fully accounted for large rotations, large displacements, initial curvatures, as well as extensionality, and contained most of the existing theories as special cases. This theory was further developed by using the mechanics-based variables to eliminate the discontinuous and sequence-dependent rotational variables[4]. Coda and Greco[5] presented a geometrically nonlinear formulation based on the position description for large deflection two-dimensional frame analysis. Then, Coda[6] proposed a solid-like finite element formulation without using the finite rotation schemes to solve the geometric nonlinear three-dimensional inhomogeneous frames. The description of the exact geometry of undeformed and deformed configurations is usually required in those theories, especially for naturally curved beams. Isogeometric analysis (IGA), which uses B-Splines or Non-Uniform Rational B-Splines (NURBS) instead of polynomials to describe both the exact geometry and the filed variables

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** Project supported by the National Natural Science Foundation of China (Nos. 11572132 and 11572137).

in analysis process, was first introduced by Hughes et al. in 2005[7], and was rapidly extended to different fields of computational mechanics, engineering and sciences[8–11]. It has already been noted that increasing the accuracy of the geometry representation entails a significant increase in the accuracy of the result[12,13]. One of the major advantages of NURBS is that it can exactly represent all conic sections[7,14] and there exist many efficient and numerically stable algorithms to generate the NURBS objects[15]. Another major advantage is that the concept of k-refinement allows the use of basis functions of higher continuity[14]. Several studies have been performed to develop the one-dimensional structural elements by using the concept of IGA in the last few years[16–20], where the locking phenomena were mainly concerned. Weeger et al.[21] analyzed the vibration of nonlinear Euler-Bernoulli beams with von Kármán strain-displacement relationship by means of the isogeometric finite elements. Based on the Green-Lagrange strain measure, Raknes et al.[22] developed an isogeometric rotation-free cable formulation. Cazzani et al.[23] presented a plane-curved beam based on the Timoshenko model and NURBS. Lan et al.[24] integrated the NURBS geometry and the rational absolute nodal coordinate formulation for the dynamic analysis of flexible bodies.

This paper attempts to establish a planar rotation-free Euler-Bernoulli beam model and focuses on the snap-back phenomena of flexible structures. In §II, a flexible beam model based on the position description is presented with the Euler-Bernoulli assumption. In §III, the isogeometric finite element discretization is completed. Then, several typical examples with geometric nonlinearities are used to demonstrate the accuracy and effectiveness of the proposed formula in §IV.

II. Description of Planar Beam Structure

We consider a naturally curved planar-beam, as is shown in Fig.1. Three coordinate systems are adopted to describe the initial and current configurations. The ab system is a right-hand inertial Cartesian coordinate system used for the reference configuration; the xy system is an orthogonal curvilinear coordinate system with the x axis connecting the reference point of each cross-section of the initial beam; and the $\xi\eta$ system is a local orthogonal curvilinear coordinate system attached to the current configuration. We let $\boldsymbol{i}_a$ and $\boldsymbol{i}_b$ denote the unit direction vectors of the ab coordinate system; ${}^0\boldsymbol{i}_1$ and ${}^0\boldsymbol{i}_2$ denote the unit direction vectors of the xy coordinate system; and ${}^t\boldsymbol{i}_1$ and ${}^t\boldsymbol{i}_2$ denote the unit direction vectors of the $\xi\eta$ coordinate system. Moreover, $s \in [0, L] \subset \mathrm{R}$ is the initial arc-length along the x axis from the end of the beam to the observed reference point, and $L \in \mathrm{R}$ is the initial length of the beam.

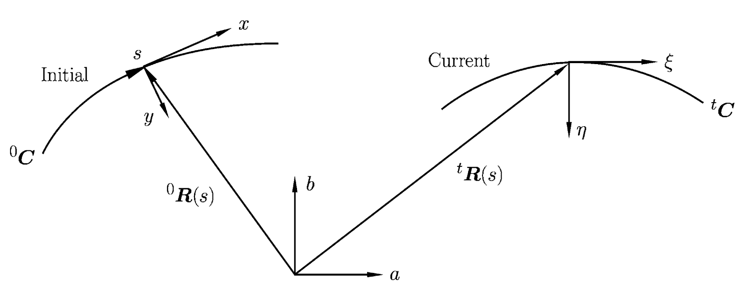

Fig. 1. Initial and current configurations of planar-beam with an arbitrary initial curvature.

2.1. Description of initial configuration

The undeformed position vector ${}^0\boldsymbol{R}$ of the reference line of the initial configuration is given by

$$ {}^0\boldsymbol{R}(s) = {}^0X(s)\boldsymbol{i}_a + {}^0Y(s)\boldsymbol{i}_b \tag{1} $$

Then, the unit tangent vector ${}^0\boldsymbol{i}_1$ of the initial reference line is defined as

$$ {}^0\boldsymbol{i}_1 = \frac{\mathrm{d}^0\boldsymbol{R}(s)}{\mathrm{d}s} = {}^0X'\boldsymbol{i}_a + {}^0Y'\boldsymbol{i}_b \tag{2} $$

Hereafter, without special statement, the prime $(\cdot)' \equiv \mathrm{d}(\cdot)/\mathrm{d}s$ denotes the derivative with respect to s. Using the orthogonality property of the tangent vector ${}^0\boldsymbol{i}_1$ and the normal vector ${}^0\boldsymbol{i}_2$, i.e. ${}^0\boldsymbol{i}_1 \cdot {}^0\boldsymbol{i}_2 = 0$, we obtain

$$ {}^0\boldsymbol{i}_2 = {}^0Y'\boldsymbol{i}_a - {}^0X'\boldsymbol{i}_b \tag{3} $$