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Wave propagation in viscoelastic phononic crystal rods with internal resonators

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A B S T R A C T

In the present work, the wave propagation in a viscoelastic phononic crystal rod with internal periodic dissipative resonators is investigated. The Kelvin-Voigt model is utilized to describe the viscoelastic behavior of host materials. The Bloch theorem is adopted to analyze the band structure of the rod. The effect of the free oscillation frequency of the resonators on the band structure is firstly studied. It is found that by tailoring the dynamic characteristics of the resonators, the coupling of the Bragg scattering (BS) and local resonance (LR) mechanisms can be harnessed to effectively widen the band gaps and enhance the wave attenuation. Then, the effects of the viscosity of the host materials and the damping of the resonators on the band structure, especially the two nearly coalescent band gaps (the first Bragg and LR ones), are investigated respectively. Furthermore, the combined effect of the two dissipative sources is also discussed. The present work is expected to be helpful to the design and applications of phononic crystals and metamaterials.

1. Introduction

Phononic crystals (PCs) are periodic structures made of two or more materials with different elastic properties. They possess frequency band gaps within which wave propagation is forbidden, independent of the wave vector [1–3]. Such band gap feature of PCs can be used in many fields, such as acoustic/elastic filters, acoustic waveguides, noise controllers, and vibration shields [4]. Mead [5] first studied the wave propagation in a periodically supported infinite beam, and revealed the band gap feature of this structure. Ruzzene et al. [6,7] applied the concept of periodic construction to a sandwich plate. They found that by placing negative Poisson's ratio core materials with different geometries periodically in the plate, the propagation of waves over specified frequency bands and in particular directions can be obstructed.

In addition, the exploitation of structural effects such as topology, geometry and local resonance has led to the development of material systems with extraordinary electromagnetic and acoustic properties, *i.e.*, so called "metamaterials" [8–10]. Taking acoustic metamaterials (AMMs) for instance, local resonance leads to a strong attenuation which has significant implications for the suppression of sound transmission [10,11] and mechanical vibrations [12–18]. The concept of local resonance has been explored to design metamaterial beams and plates with periodic resonator arrays [19–21].

Most studies on wave propagation in PCs and AMMs mainly focus on elastic medium. However, damping is an intrinsic property of materials and may have a significant influence on structural dynamic responses. To predict the performance of PCs and AMMs more accurately, it is necessary to take dissipative effects into consideration [22]. Mead [23] first investigated the effects of viscous and hysteretic damping on the wave number for a one-dimensional (1D), periodically supported beam. It is found that band gaps became less distinct if the effect of damping was considered, because complex wave numbers existed throughout the frequency spectrum. This result was reproduced qualitatively by Tassilly [24] in his study on damped PC beams. Merheb et al. [25] used the finite difference time domain method to investigate the transmission of acoustic waves in viscoleastic rubber-air PCs. Besides these researches on PCs, Manimala and Sun [26] studied the wave attenuation of 1D dissipative AMMs by considering discrete lattices involving local resonators with different types of viscous damping or hysteretic damping. Chen et al. [27] also adopted similar analysis method to study 1D viscoelastic AMMs with mass-in-mass viscous local resonators. Dissipative metamaterial plates with tunable local

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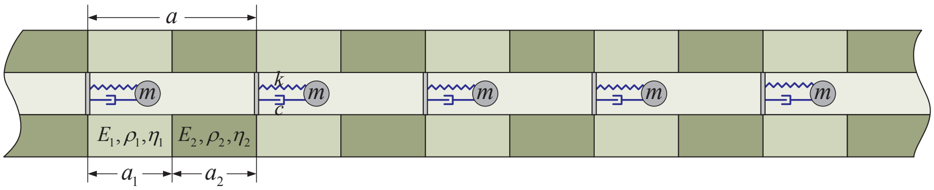

Fig. 1. Schematic of a viscoelastic phononic crystal rod with internal mass-dashpot-spring resonators, where is the lattice constant, E_1, ρ_1 and η_1 (E_2, ρ_2 and η_2) are the Young's modulus, mass density and viscosity of Material 1 (Material 2), and m, k and c are the mass, spring stiffness and damping of each resonator, respectively.

resonators [28] and two-dimensional (2D) viscoelastic metamaterials [29] were also studied by using the simple Kelvin-Voigt model. Recently, more realistic viscoelastic models, such as generalized Maxwell models, were employed to study 2D viscoelastic AMMs by Krushynska et al. [30] and Lewinska et al. [31]. A more recent work even harnessed the combined attributes of AMMs and PCs as well as viscous damping to generate the so called "metadamping" effect [32]. In all these studies, the frequency has a prescribed real value, such that the corresponding temporal parameter $\lambda = -i\omega$ is pure imaginary. This case represents a class of researches on wave propagation incorporating dissipative effects and corresponds to a steady-state situation where the medium is driven by a given frequency. Such a case is called "prescribed wave" in the present paper.

Alternatively, a wave with its amplitude decreasing with time and its frequency un-prescribed is referred to as "free wave". In this perspective, Cady [33] studied propagation of longitudinal waves in homogeneous rods. He allowed the temporal parameter λ to become complex, *i.e.*, $\lambda = -\xi\omega \pm i\omega_d$, where ω_d was the damped propagation frequency, and $-\xi\omega$ was the temporal attenuation constant. Mukherjee and Lee [34] investigated wave propagation in one-dimensional PCs, and developed the dispersion relations for both the damped frequency and the temporal attenuation constant. Hussein [35] presented the band structure in the Brillouin zone and the damping ratio corresponding to each Bloch wave, and demonstrated that the damping qualitatively altered the shape of the dispersion curve. They also extended their analysis to the study of a 2D PC, and revealed intriguing phenomena such as branch overtaking and branch cut-off [36]. Sprik and Wegdam [37] analyzed the propagation of sound waves in three dimensional periodic lattices of solid-solid and solid-liquid composites. Zhang et al. [38] investigated the absolute acoustic band gaps for two-dimensional periodic arrays of silica cylinders in viscous liquid, and found that when the viscous penetration depth was comparable to the structural length scale, the structures possessed large absolute acoustic band gaps comparing with those without viscosity. It is also noted that Andreassen and Jensen [39] analyzed the bandgap of a 2D dissipative PC for both the two cases of free and prescribed wave propagation, and pointed out that comparable results are predicted for small to medium amounts of material dissipation and for long-wavelength waves.

Through the above literature review, it is found that lots of works have been conducted on wave propagation in viscoelastic PCs and AMMs. Although wave attenuation of viscoelastic PCs with internal dissipative resonators has been considered by some researchers such as Krushynska et al. [30] and DePauw et al. [32], the coupling of Bragg scattering (BS) and local resonator (LR) mechanisms in such kind of dissipative systems, or phononic resonators as termed by DePauw et al. [32], has not been completely clarified yet. In order to at least partly address this problem, the wave propagation of a 1D viscoelastic PC rod with internal dissipative resonators is studied in the present work, with both the two cases of free and prescribed wave propagation taken into account. It is found that for both non-dissipative (purely elastic) and dissipative PC rods, the BS-LR coupling can be harnessed to effectively widen the two nearly coalescent band gaps (the Bragg and LR ones, respectively) and enhance the attenuation via tailoring the dynamic characteristics of the resonators. The effects of two dissipative sources (the viscosity of the host materials and the damping of the internal resonators) on the BS-LR coupling and their significant implications to wave propagation are also discussed in detail.

The rest of the present work is structured as follows. In Section 2, the governing equations for each part of the PC rod as well as the internal resonator within each unit cell are firstly derived, and then by employing the Bloch theorem, two eigenvalue problems are formulated respectively in Subsections 2.1 and 2.2 for cases of prescribed and free wave propagation. The band structures of the PC rod with internal resonators under both the non-dissipative and dissipative conditions are presented in Section 3, and the effects of the free oscillation frequency, mass, and damping of the internal resonators as well as the viscosity of the host materials are discussed in detail. At last, some conclusions are drawn in Section 4.

2. Formulation

As shown in Fig. 1, a viscoelastic PC rod with periodic internal resonators is considered. The rod is composed of repetition of alternating Material 1 with length a_1 and Material 2 with length a_2. The lattice constant is denoted by a which is equal to $a_1 + a_2$. The origin of the local coordinate is located at the junction of Material 1 and Material 2. The internal resonators, each with mass m, spring stiffness k and damping c, are fixed at the left end of each unit cell periodically.

The motion equation of the rod in the longitudinal direction reads:

$$\frac{\partial\sigma^{(r)}}{\partial x}A + f^{(r)} = \rho_r A\frac{\partial^2 u^{(r)}}{\partial t^2}, \tag{1}$$

where $r = 1,\ 2$ is used for distinguishing Material 1 from Material 2, $\sigma^{(r)} = \sigma^{(r)}(x, t)$, A, $f^{(r)} = f^{(r)}(x, t)$, ρ_r, and $u^{(r)} = u^{(r)}(x, t)$ denote the stress, the cross-sectional area of the rod, the external force, the mass density, and the longitudinal displacement, respectively. As indicated, all these quantities, except for the cross-sectional area and the mass density, are dependent of the position x and the time t.

Due to the diversity and complexity of dissipative mechanisms, the development of a universal damping model stands as a major challenge. The simple Kelvin-Voigt model, which consists of a spring and a dashpot connected in parallel, is widely used to describe the time-dependent property of viscoelastic medium [40–44]. It should also be noted that due to the lack of enough fitting parameters, the Kelvin-Voigt model may not fit well with experimental data. To solve this problem, some more sophisticated and experimentally validated models, such as the generalized Kelvin-Voigt model or the generalized Maxwell model, could be used instead. However, via comparison with a validated generalized Maxwell model, Krushynska et al. [30] pointed out that the Kelvin-Voigt model provides reliable results concerning the wave dispersion, except in a very low frequency range which is significantly below band gaps and not of a critical importance. Considering this reason and for the sake of simplicity, the Kelvin-Voigt model is adopted here to describe the viscoelastic behavior of the host materials, so as to focus on the effect of viscosity on the coupling of the BS and LR mechanisms. According to the Kelvin-Voigt model, the constitutive relation reads:

$$\sigma^{(r)} = E_r\frac{\partial u}{\partial x} + \eta_r\frac{\partial^2 u}{\partial x\partial t}, \tag{2}$$

where E_r is the Young's modulus, and η_r is the viscosity. In the absence of body forces, substituting Eq. (2) into Eq. (1) yields the motion equation of the viscoelastic PC rod: