Applied Acoustics 140 (2018) 315–326

Contents lists available at ScienceDirect

Applied Acoustics

journal homepage: www.elsevier.com/locate/apacoust

Effects of the thickness on the stability of axially moving viscoelastic rectangular plates

Mouafo Teifouet Armand Robinson[a,*], Sarp Adali[b]

[a] Department of Physics, Faculty of Science, University of Dschang, Cameroon, P.O. Box 67 Dschang, Cameroon
[b] Discipline of Mechanical Engineering, University of KwaZulu-Natal, Durban 4041, South Africa

ARTICLE INFO

Keywords:
Instability
Viscoelastic plate
Non-uniform thickness
Differential quadrature method

ABSTRACT

The differential quadrature method is employed to solve the differential equation governing the in-plane moving rectangular viscoelastic plate with concave or convex cross sections. The boundary conditions of the plate are specified as simply supported-free (SFSF) and clamped-simply supported (CSCS). Plots showing the influence of the cross section's form on the plate's stability are presented and it is observed that the plate generally undergoes divergence instability but coupled mode instability also occurs for SFSF boundary conditions. Coupled mode instability appears when the cross-section is convex or varies linearly. Also, the increase of the thickness ratio plays an increasing role on the critical instability values of the moving speed.

1. Introduction

Plates with varying thicknesses have shown their great usefulness due to their potential applicability in many engineering domains. For example, varying thickness can lead to the design of lighter structures or reduce the quantity of material used in their manufacture. The above mentioned considerations explain why plates with non-uniform thicknesses have been studied extensively [1–6].

Moving plate-type structures may lose stability during their manufacturing process or when they are operating due to the increase in the moving speed. Literature indicates that such structures may undergo dynamic or static instability depending on the applicable boundary conditions [7–10].

Viscoelasticity itself appears to play a great role on the nature of the instability that may occur during the moving process of plates [11–13]. Meanwhile, axially moving viscoelastic plates with varying thickness have not yet been studied extensively according to the existing literature. The only existing previous work is Zhou and Wang [14] who studied the moving viscoelastic plate with the thickness varying parabolically in the y-direction. The effects of the thickness ratio on the stability of SSSS and CSCS plates were carried out in detail [14]. To the authors' best knowledge, there is presently no work involving the study of the influence of both concave and convex cross sections on the stability of the axially moving viscoelastic plates.

The differences between our work and that of [14] is that the thickness in the present study varies in the x-direction, leading therefore to a different differential equation. Consequently, the instability of the plate is qualitatively and quantitatively different from what was observed in the previous studies. Here three kinds of non-uniform thicknesses are considered whereas in [14], only a plate with concave cross section was considered.

Moreover there is no work in the literature studying the stability of axially moving viscoelastic plates with varying thickness using differential quadrature method considering free edges. That absence can be due to the complicated form of the governing differential equation or the implementation of free edges using differential quadrature method.

The main aim of the present work is to derive the equation of axially moving viscoelastic plates with linear, concave and convex cross sections in the x-direction by using the Kelvin-Voigt law of viscoelasticity. The differential quadrature scheme is then employed to solve the corresponding fourth order differential equation. We first compare our results with those available in the literature to verify the convergence of DQM by using two different boundary conditions which are Simply supported (SSSS) and clamped (CCCC). Afterward, the equation to compute the complex eigenvalues of the problem is analyzed and the effects of cross sectional shapes on the stability of the system are studied for CSCS and SFSF boundary conditions.

2. Equation of motion of axially moving viscoelastic plate with varying thickness

The theoretical equation of vibration of axially moving viscoelastic plate with varying thickness is established in this section. The rectangular plate has dimensions a and b along x and y directions,

* Corresponding author.
E-mail address: armandmouafo@yahoo.fr (M.T.A. Robinson).

https://doi.org/10.1016/j.apacoust.2018.05.005
Received 6 November 2017; Received in revised form 10 February 2018; Accepted 3 May 2018

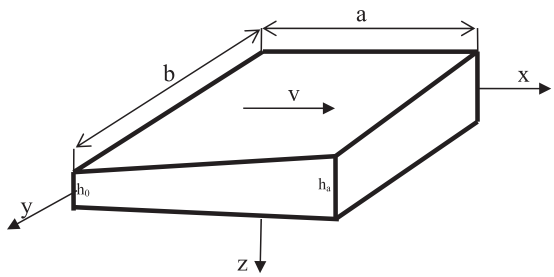

Fig. 1. Axially moving viscoelastic plate with linearly varying thickness.

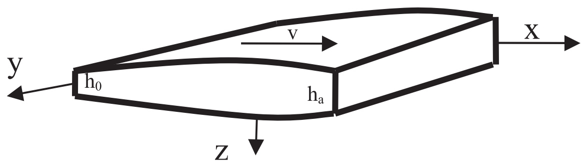

Fig. 2. Axially moving viscoelastic plate with concave cross section.

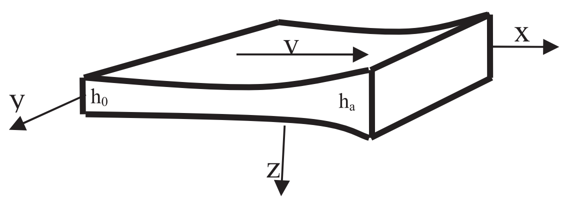

Fig. 3. Axially moving viscoelastic plate with convex cross section.

respectively, and varying thickness $h(x)$ along the z axis. h_0 and h_a are thicknesses at $x = 0$ and $x = a$, respectively. Three cases of non-uniform thickness are considered, namely, linearly varying thickness shown in Fig. 1, concave and convex cross sections presented in Figs. 2 and 3. The Young's modulus of the material is denoted by E, its Poisson's ratio by ν and density by ρ.

The strain-displacement relation according to the Kirchhoff plate theory is as follows:

$$\varepsilon_x = -z\frac{\partial^2 w}{\partial x^2}, \quad \varepsilon_y = -z\frac{\partial^2 w}{\partial y^2}, \quad \varepsilon_{xy} = \frac{\gamma_{xy}}{2} = -z\frac{\partial^2 w}{\partial x \partial y} \tag{1}$$

where u, v, w are displacement components in the x, y and z directions, respectively, ε_x and ε_y are the normal strain components, and γ_{xy} is the shear strain component.

The plate obeys the Kelvin-Voigt constitutive behavior of viscoelasticity leading to the following stress–strain relations [15–16];

$$\mathbf{s}_{ij} = 2G\mathbf{e}_{ij} + 2\eta\dot{\mathbf{e}}_{ij}, \quad \sigma_{ii} = 3K\varepsilon_{ii} \tag{2}$$

where K, η, G denote the bulk modulus, viscoelastic coefficient and shear modulus, respectively. Their expressions are $K = E/3(1-2\nu)$ and $G = E/(1 + 2\nu)$. $\mathbf{s}_{ij}$ and $\mathbf{e}_{ij}$ represent the deviatoric tensors of stress and strain, respectively, while $\mathbf{s}_{ii}$ and σ_{ii} are, respectively, the spherical tensors of strain and stress. Let (M_x, M_y) be the bending moments and (M_{xy}, M_{yx}) the twisting moments. They are linked to the stress by the following relations:

$$M_x = \int_{-h(x)/2}^{h(x)/2} z\sigma_x dz, \quad M_y = \int_{-h(x)/2}^{h(x)/2} z\sigma_y dz \tag{3a}$$

$$M_{xy} = \int_{-h(x)/2}^{h(x)/2} z\sigma_{xy} dz, \quad M_{yx} = \int_{-h(x)/2}^{h(x)/2} z\sigma_{yx} dz \tag{3b}$$

where σ_x and σ_y are the normal stress components, and σ_{xy} and σ_{yx} are the shear stress components. Considering that the plate is moving with a speed ϑ, the equation governing the vibrations of axially moving viscoelastic plate with non-uniform thickness, in terms of moments, is given by

$$\frac{\partial^2 M_x}{\partial x^2} + 2\frac{\partial^2 M_{xy}}{\partial x \partial y} + \frac{\partial^2 M_y}{\partial y^2} - \rho h(x)\left(2\vartheta\frac{\partial^2 w}{\partial t \partial x} - \vartheta^2\frac{\partial^2 w}{\partial x^2} - \frac{\partial^2 w}{\partial t^2}\right) = 0 \tag{4}$$

After performing a Laplace transformation of Eq. (4), one gets a new equation with Laplace transformations of moments. Taking into account Eqs. (1) and (3), and the transformations performed in [15–18], the differential equation governing the vibration of the axially moving viscoelastic plate with non-uniform thickness is obtained as:

$$\begin{aligned}
&\frac{h^3(x)}{12}\left(A_3 + A_4\frac{\partial}{\partial t} + A_5\frac{\partial^2}{\partial t^2}\right)\nabla^4 w + \frac{6h^2(x)}{12}\frac{dh(x)}{dx}\left(A_3 + A_4\frac{\partial}{\partial t}\right.\\
&\quad\left. + A_5\frac{\partial^2}{\partial t^2}\right)\frac{\partial}{\partial x}\nabla^2 w + \left[\frac{3h^2(x)}{12}\frac{d^2h(x)}{dx^2}\right.\\
&\quad\left. + \frac{6h(x)}{12}\left(\frac{dh(x)}{dx}\right)^2\right]\left[\left(A_3 + A_4\frac{\partial}{\partial t} + A_5\frac{\partial^2}{\partial t^2}\right)\frac{\partial^2 w}{\partial x^2}\right.\\
&\quad\left. + \left(A_6 + A_7\frac{\partial}{\partial t} - A_5\frac{\partial^2}{\partial t^2}\right)\frac{\partial^2 w}{\partial y^2}\right] + \rho h(x)\left(A_1 + A_2\frac{\partial}{\partial t}\right)\left(2\vartheta\frac{\partial^2 w}{\partial t \partial x}\right.\\
&\quad\left. + \vartheta^2\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial t^2}\right) = 0
\end{aligned} \tag{5}$$

where

$$\begin{aligned}
&A_1 = 3K + 4GA_2 = 4\eta, \quad A_3 = 4G(3K + G), \quad A_4 = 4\eta(2G + 3K),\\
&A_5 = 4\eta^2 A_5 = 4\eta^2 \quad A_6 = 2G(3K - 2G), \quad A_7 = 6K\eta - 8G\eta
\end{aligned} \tag{6a}$$

$$\nabla^4 w = \frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4}, \quad \nabla^2 w = \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \tag{6b}$$

In order to facilitate the computational process, the following the dimensionless variables are introduced

$$X = \frac{x}{a}, \quad Y = \frac{y}{b}, \quad \overline{w} = \frac{w}{h}, \quad \lambda = \frac{a}{b}, \tag{7a}$$

$$V = \frac{a}{h}\sqrt{\frac{12\rho(1-\nu^2)}{E}}\vartheta, \quad \tau = \frac{th}{a^2}\sqrt{\frac{E}{12\rho(1-\nu^2)}}, \quad \sigma = \frac{h}{a^2}\sqrt{\frac{E}{12\rho(1-\nu^2)}}\frac{\eta}{E} \tag{7b}$$

Introducing Eq. (7) into Eq. (5) leads to the dimensionless form of the governing equation of the axially moving viscoelastic plate with varying thickness. Since three different profiles of non-uniform plate will be used, three equations are derived accordingly after expressing the transverse displacement as:

$$\overline{w}(X, Y, \tau) = W(X, Y)\exp(\sqrt{-1}\,\omega\tau) \tag{8}$$

where ω is the dimensionless complex frequency number. Considering Eqs. (5)–(8) and the corresponding form of the non-dimensional thickness $h(X) = h(x)/h_0$, the equations of motion are derived as follows:

2.1. Linearly varying thickness

In this case, $h(X) = 1 + (h_a/h_0 - 1)X$ and the dimensionless governing equation is: