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# Band gap and double-negative properties of a star-structured sonic metamaterial

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ABSTRACT

Sonic metamaterials have a wide range of applications in wave control and super-resolution imaging, and are favored for their several unique and advantageous properties. However, current double-negative sonic metamaterials have complex structures composed of various materials, which limits their design and application. Thus, we must produce double-negative features using a simple structure of one material. Because of their unique concave configurations and various resonances, star-shaped structures readily form band gaps and show superior material properties. In this study, we designed and simulated star-shaped single-phase metamaterials, considered ideal structures. Our numerical results suggest that these metamaterials have two band gaps as well as double-negative properties over specific frequency ranges. Moreover, we investigated how their band gap and double-negative properties depended on the concave angle.

## 1. Introduction

Sonic metamaterials [1] are artificial composite materials with acoustic characteristics not found in natural materials, such as negative effective mass density [2–4] and negative effective modulus [5,6]. Negative effective mass density means that the direction of acceleration and driving force of the medium are opposite under dynamic pressure; a negative effective modulus can be interpreted as a scattering field generated by a resonance that is much larger than the incident field, producing contrary changes in pressure and volume. Having both negative effective modulus and density, sonic metamaterials are compelling because of their negative refraction over a certain frequency range [7] as well as the inverse Doppler effect [8] and super-resolution imaging [9–14].

Generally, sonic metamaterials gain double-negative properties by being composed of materials with different elastic constants, which forms hybrid resonances by accessing the various resonance modes of the structural units [15]. Thus, double-negative acoustic metamaterials often have complex structures and must be composed of multiple materials. However, such structures are difficult to design and use. Thus, it is important to study how to produce double-negative properties using a single-phase material in a simple structure in order to better design and develop acoustic devices made of metamaterials.

Using single-phase materials to achieve the double-negative property is challenging because this extraordinary property depends on the vibration characteristics of the artificial "atomic" (the sub-wavelength locally resonant units) in a specific frequency range. Producing double-negativity requires introducing a resonant unit with both dipole and monopolar resonances in its structure [15–20]. Negative mass is usually achieved by introducing a lumped mass into the sub-wavelength structure constituting the spring resonant unit [16]; the units show negative effective density at the dipole resonance frequency. To produce a negative effective modulus, a Helmholtz resonator or a rotating resonator [5,21] is usually introduced into the sub-wavelength structure, producing a negative effective modulus near the monopolar resonance frequency. Hybridization states and chiral structures are also used in designing double-negative acoustic metamaterials [22,23].

Based on the above considerations, Liu et al. produced double-negative metamaterials by introducing a chiral structure, which had a negative group velocity over specific frequency ranges [22,23]. Using four resonant units, Lai et al. formed a hybridized elastomeric solid and produced a two-dimensional double-negative metamaterial based on hybrid resonance states [15]. Yang et al. achieved double-negative properties in a double-layer film system [19]; if the double membranes vibrate in phase, the dipole vibration provides a negative effective mass density, whereas if the double membranes vibrate in opposite phase, the monopole vibration provides a negative effective modulus. However, these double-negative metamaterials have multiple material

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phases, and their complex structures are difficult to use. Although Zhu et al. developed a single-phase metamaterial with a chiral microstructure—which can achieve both negative effective mass density and modulus owing to simultaneous translational and rotational resonances—the structure is also complicated and bulky. Thus, it is necessary to introduce new resonance mechanisms containing both dipole and monopole resonances in a lightweight structure if single-phase metamaterials are to be developed.

Materials with star-shaped structures possess a negative Poisson's ratio from its special concave configuration. Compared with traditional materials, the star-shaped structure has higher energy absorption efficiency, better designability, homodromous bending capability, and low mass [24–26]. Its mechanical properties can be adjusted by adjusting its geometry, such as its thickness, beam length, and beam concavity. Considering their wave characteristics, with volume changes and various resonance behaviors, the star-shaped structure is good at forming a low-frequency band gap and extraordinary properties because of its concave configuration. In this paper, we systematically study the formation mechanism of band gaps in star-shaped sonic metamaterials. Revealed by numerical analysis, this star-shaped structure shows special resonance properties, achieves wide band gaps, and has the double-negative property in a specific frequency range. We also investigate how its geometry affects its band gap, as described below.

## 2. Star-shaped structure model and numerical calculation method

Depending on its cell structure, star-shaped structures are typical auxetics that come in either a four-pointed or six-pointed star [25,26]. Its structure is made of slender beams, so it is very light and enables a variety of resonances. For example, the presence of a lumped mass in the concave node produces a dipole resonance, while bending of the cant beams and rotation of the point mass induces a monopolar resonance. These resonances enable the superior acoustic properties such as negative effective density and negative effective modulus. In this paper, we focus on the wave characteristics of the four-pointed star, the unit cell of which is given in Fig. 1.

Although the thickness of the structure will affect the position of its band gap, we only studied two-dimensional structures in this paper [27]. This structure consists of four straight beams of equal length $L_1$ and eight star-shaped cant beams of equal length $L_2$. The beams have

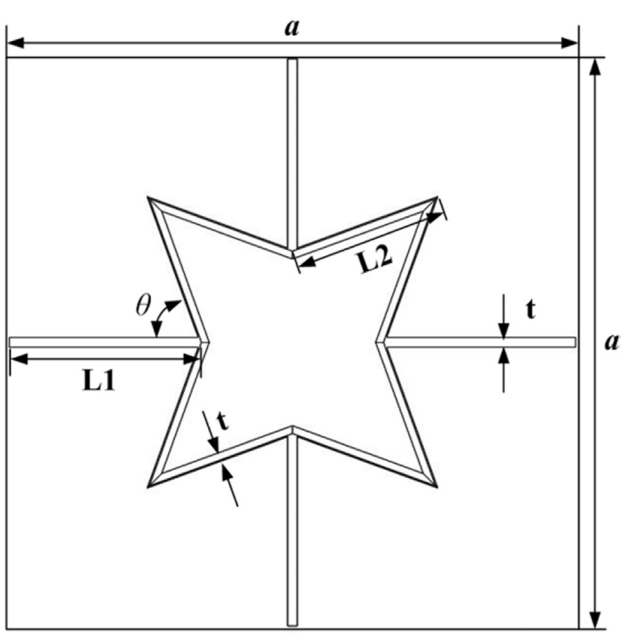


**Fig. 1.** Schematic of the unit four-pointed star structure.

equal thickness $t$. The counterclockwise angle between adjacent cell walls is denoted as $\theta$. The unit cells are arranged in a square lattice with the lattice constant $a$, which can be expressed as $2\left\{\frac{\sin(\theta-45^\circ)}{\sin(45^\circ)}\cdot L_2 + L_1\right\}$. In the calculations, we set $L_1$, $L_2$, and $t$ to 0.5 cm, 1 cm, and 0.05 cm, respectively. When $L_1$, $L_2$, and $t$ are constant, the angle $\theta$ of the star-shaped structure must be more than 45° and less than 135°. The whole structure is given the material properties of steel.

The band structure, vibration modes, and effective medium parameters of the star-shaped structure were all studied using *COMSOL Multiphysics* finite-element method (FEM) software. For two-dimensional sonic metamaterials, the displacement can be divided into an in-$x$, $y$ plane and an out-$x$, $y$ plane. We only considered the fluctuations in the $\underline{x\text{-}y}$ plane in this study. In the calculations, periodic boundary conditions are imposed to simplify the computational domain to one unit cell.

$$u(x,y+a) = u(x,y)e^{i(k_y \cdot a)},$$
$$v(x,y+a) = v(x,y)e^{i(k_y \cdot a)} \tag{1}$$

Based on Bloch's theorem, solving the $k$–$w$ relations on the boundary of the Brillouin zone yields the dispersion relation for the whole structure [28,29]. When calculating transmission loss, the displacements of the finite elements are taken in one direction only; the periodic boundary conditions account for an infinite array of units in the other direction. Assuming that the incident plane wave is on the surface of the structure, we determine the transmission loss of the star-shaped structure from the transmission coefficient. Meanwhile, to reduce the reflected wave at the boundary, a perfect matching layer is used in the incident direction. Under the long-wave hypothesis, the acoustic metamaterials can be regarded as homogeneous, with special properties according to the equivalent medium theory. Also, the dynamic behavior can be represented using equivalent medium parameters to determine the propagation of the acoustic wave. The effective parameters of the medium were calculated by determining the displacement, strain, stress, and force on the boundaries; the details of our calculation methods can be found elsewhere [15,30].

The effective mass was calculated according to Newton's second law:

$$\rho^{eff} = \frac{m^{eff}}{a^2} = \frac{F_x^{eff}}{\ddot{u}_x^{eff}a^2} = \frac{=-F_x^{eff}}{\omega^2 u_x^{eff}a^2} \tag{2}$$

where $C_{11}^{eff}$, $C_{12}^{eff}$, and $C_{44}^{eff}$ are the effective stiffness tensors; $T_{xx}^{eff}$, $T_{yy}^{eff}$, and $T_{xy}^{eff}$ are the xx, yy, and xy components of effective stress tensor. $S_{xx}^{eff}$, $S_{yy}^{eff}$, and $S_{xy}^{eff}$ are xx, yy, and xy components of the effective strain tensor. Under external stimulation, there are three unknowns in the constitutive relations: $C_{11}^{eff}$, $C_{12}^{eff}$, and $C_{44}^{eff}$. The others can be obtained from the stress and deformations of unit boundary.

$T_{xx}^{eff}$, $T_{yy}^{eff}$, and $T_{xy}^{eff}$ were calculated as follows:

$$T_{xx}^{eff} = \frac{\int T_{xx}dy|_{x=0} + \int T_{xx}dy|_{x=a}}{2a}$$
$$T_{yy}^{eff} = \frac{\int T_{yy}dx|_{y=0} + \int T_{yy}dx|_{y=a}}{2a}$$
$$T_{xy}^{eff} = \frac{\int T_{xy}dx|_{y=0} + \int T_{xy}dx|_{y=a}}{2a} \tag{3}$$

.

$S_{xx}^{eff}$, $S_{yy}^{eff}$, and $S_{xy}^{eff}$ were calculated as follows:

$$S_{xx}^{eff} = \frac{\int u_x dy|_{x=a} - \int u_x dy|_{x=0}}{a^2}$$
$$S_{yy}^{eff} = \frac{\int u_y dy|_{x=a} - \int u_y dy|_{x=0}}{a^2}$$
$$S_{xy}^{eff} = \frac{\int u_x dx|_{y=a} - \int u_x dx|_{y=0} + \int u_y dy|_{x=a} - \int u_y dy|_{x=0}}{2a^2} \tag{4}$$

The effective bulk modulus $\kappa^{eff}$ and effective shear modulus $\mu^{eff}$ were defined by $C_{11}^{eff}$, $C_{12}^{eff}$, and $C_{44}^{eff}$: