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A signal-adapted wavelet design method for acoustic emission signals of rail cracks

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ABSTRACT

Though multiresolution analysis has found successful applications on Acoustic emission (AE) signals, existing wavelets suffer from the problem of expressing AE signals in many applications. In the field of rail defect detection, complicated AE signals emitted from rail crack growth cannot be described properly by commonly used wavelets. In order to give better expressions of these signals, this paper proposes a design approach for signal-adapted wavelet in the frame of a two-band analysis/synthesis system, in which the problem of wavelet design is converted into a constrained optimization problem. For perfect reconstruction of the system, necessary and sufficient conditions, as well as orthogonality and normalization of the wavelet, are considered as constraints of the optimization problem. In order to preserve more information of crack growth AE signals, Euclid norm of the error between the approximation signal and the input signal is minimized in the cost function. AE signals of crack growths from a tensile test of a steel specimen are acquired for experimental verification. By comparing with the commonly used wavelets, it is demonstrated that the wavelet designed by the proposed method has superior performance in expressing the crack AE signal, and can outperform the most suitable existing wavelet due to its adaptivity. Moreover, the designed wavelet shows good robustness against noise, which has profound meaning for rail crack detection in practical applications.

1. Introduction

Acoustic emission (AE) technique has shown superior performance in nondestructive detection, because of its sensitivity and the rich information contained in the AE signals. Since AE detection directly takes advantage of the signal generated from the physical deformation of materials, it has promising applications in the area of rail defect detection [1–4]. Of all forms of rail defect, rail crack growth is a fatal kind, because it will lead to catastrophic rail fracture in a short time, especially in high-speed railway [5,6]. Therefore, AE signals emitted from rail cracks have received considerable attention. Because the crack AE signals are transient and impulsive, their abundant information in time and frequency domains should be made full use of. From this point of view, multiresolution analysis is an effective tool for AE signal analysis. Among all multiresolution analysis methods, wavelet transform is widely adopted due to its good performance and easy implementation [7,8]. However, because of complicated mechanics of steel deformation and rapture, generation of crack growth will lead to various AE signals. Existing wavelets cannot express these signals well enough due to their irrelevances to the objective signal. Thus, in order to provide better expression of crack AE signals for rail crack detection by multiresolution methods, designing of signal-adapted wavelets becomes a viable approach.

Since Mallat algorithm [9] put wavelet transform and its inverse transform into a frame of a two-band analysis/synthesis system, filter banks design has been the most applicable method for wavelet design. In this context, wavelet construction can be accomplished by designing the filter banks, which has easy implementation and low calculation. Therefore, in this paper, designing of filter banks is utilized to construct our own wavelet. On this basis, there are two orientations to design signal-adapted wavelets. One idea is to improve the existing wavelets under the instruction of desired properties. For example, [10] lifted the old multiresolution analysis to a new one, in which desirable vanishing moment was realized by introducing a corresponding partitioning. [11] constructed a new wavelet on the basis of Mexican-hat function. However, in practice, there is an intrinsic drawback of this kind of methods, i.e. these methods are severely constrained by the initial choice of multiresolution analysis.

A more widely accepted idea of signal-adapted wavelet design is transferring the problem into a constrained optimization problem, in which objective properties can be expressed by linear or nonlinear constraints and "adapt" can be achieved by maximizing or minimizing a

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certain cost function. For example, perfect reconstruction (PR) conditions can be formulated by parameters of the filters to get a distortion-free output of the two-band analysis/synthesis system [12]. For removing the redundancy from coefficient calculation, orthogonality property can be expressed by the relation between the filters [13]. In order to construct an energy normalized wavelet with good localizing feature [14], normalization and compact support constraints can be formulated using the parameters of the filters as well. In our situation, PR conditions, orthogonality, and normalization are all considered. Moreover, since the crack AE signals are transient, aperiodic signals without symmetric property, symmetry property can be excluded from our constraints. Moreover, in order to allow for more freedom in designing our signal-adapted wavelets, constraints of regularity and vanishing moment are neglected.

With regard to “adapt”, researches were made from various aspects. For one kind, wavelet coefficients are taken advantage of. Since coefficients are the signal’s projection on the approximation subspace, this kind can be called space-partitioning methods. For example, [15] was inspired by the idea of sharpening filter used in image enhancement. In [16], the signal’s projection on the detail subspace was minimized and the approximation coefficients were maximized to boost the smoothness of the multiresolution analysis. Expect for the wavelet coefficients, reference signals can also be introduced to highlight some specific features. As an example, [17] minimized the difference between the reference signal and the desired one to obtain the matched wavelet. Apart from these, intrinsic characteristics of multiresolution analysis such as the dilation equation were made use of in [18]. In addition, wavelet design method in the frequency domain was proposed in [19]. While [20,21] started from the point of systemic matching and minimized the coding gain related cost function to obtain the optimized wavelet. From a statistical point of view, [22] found the relation between the wavelet variance and Allan variance of a Gaussian-Markov process, then constructed an adaptive wavelet statistically. To conclude, the wavelet and the signal of interest can be matched from various perspectives. In our case, since AE signals contain a lot of physical information of the cracks, it is desirable that the signal of interest is reconstructed as much as possible. Here, the approximation signal reconstructed from the low-pass filter is assumed to be the closest approximation of the input signal, so the difference between them should be as small as possible. Thus, the signal’s projection on the approximation subspace is maximized. Therefore, in order to preserve the information of crack AE signals, the error of the approximation signal and the desired signal is minimized to obtain a signal-adapted wavelet.

In this paper, to design signal-adapted wavelets for crack AE signals, we use a two-band analysis/synthesis system with four finite impulse response (FIR) filters. A constrained optimization problem is established to acquire the optimal parameters of the filters. Necessary and sufficient conditions for PR, as well as orthogonality and normalization of the wavelet, are considered to formulate the constraints. Then, for precise expression of the AE signal, the error of the approximation signal and the desired signal is utilized to construct the cost function. After the constraints and cost function are determined, the signal-adapted wavelet is obtained through optimizing. The paper is organized as follows. In Section 2, the signal-adapted wavelet design method is proposed, where constraints and the cost function of the optimization problem are specified. Section 3 gives a brief introduction of the experiments, from which crack AE signals for investigation are acquired. In Section 4, performances of the wavelets designed by the proposed method are experimentally verified through comparison. Finally, conclusions are made in Section 5.

2. Design of signal-adapted wavelet for crack AE signal

In the frame of a two-band analysis/synthesis system, the aim of “adapt” is transferred into a constrained optimization problem with regard to parameters of the filters. Here, constraints from three aspects

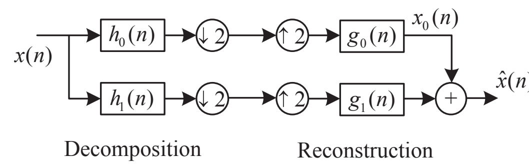

Fig. 1. Two-band analysis/synthesis system.

are considered. Firstly, necessary and sufficient conditions for PR are derived in frequency domain, in which aliasing, amplitude, and phase distortions are avoided effectively. Secondly, orthogonal condition is formulated and the intrinsic relation between PR and orthogonality is revealed. Moreover, wavelets are normalized by the normalization constraint. Then, a cost function is proposed to minimize the information loss of the wanted signal in the process of wavelet transform. At last, a constrained optimization problem of signal-adapted wavelet design is constructed.

2.1. Orthogonal PR constraints of a two-band FIR analysis/synthesis system

2.1.1. Perfect reconstructed two-band FIR analysis/synthesis system

Since Mallat [9] has put wavelet transform into the frame of a two-band analysis/synthesis system, wavelet construction is accomplished by designing of the filters. As shown in Fig. 1, wavelet decomposition is achieved by filtering the input signal $x(n)$ using low-pass analysis filter $h_0(n)$ and high-pass analysis filter $h_1(n)$. The reconstruction signal $\hat{x}(n)$ is obtained by adding up the filtering results from low-pass synthesis filter $g_0(n)$ and high-pass synthesis filter $g_0(n)$. In Fig. 1, decimators and interpolators keep the calculation amount of each decomposition/reconstruction step independent of the transform level [23]. However, they also cause side effects such as aliasing and image effects that lead to significant difference between $x(n)$ and $\hat{x}(n)$. In our case, the wavelet transform is assumed to preserve all information of the original signal without any leakage. Therefore, it is important to make sure that the analysis/synthesis system is PR guaranteed. Here, assuming filters in the analysis/synthesis system are FIR filters, necessary and sufficient PR conditions of FIR analysis/synthesis filter banks are derived as follows.

With z transform, down-sampling the input $X(z)$ results in a new signal with z-transform $Y(z)$, where

$$Y(z) = \frac{1}{2}[X(z^{1/2}) + X(-z^{1/2})].$$

Moreover, up-sampling by 2 corresponds to replacing z by z^2 in the z transform [23]. It is clear that output of the analysis/synthesis system in Fig. 1 can be obtained as follows:

$$\begin{aligned} X(z) &= \tfrac{1}{2}[G_0(z)\quad G_1(z)]\begin{bmatrix} H_0(z) & H_0(-z) \\ H_1(z) & H_1(-z)\end{bmatrix}\begin{bmatrix} X(z) \\ X(-z)\end{bmatrix} \\ &= \tfrac{1}{2}[H_0(z)G_0(z) + H_1(z)G_1(z)]X(z) + \tfrac{1}{2}[H_0(-z)G_0(z) \\ &\quad + H_1(-z)G_1(z)]X(-z) \end{aligned}$$

where $H_0(z)$, $H_1(z)$, $G_0(z)$, $G_1(z)$ are the z transforms of the corresponding analysis filters and synthesis filters. In addition, $H_0(z) = h_0(0) + h_0(1)z^{-1} + h_0(2)z^{-2} + \cdots + h_0(N-1)z^{-(N-1)}$, in which N is the filter length. And the above 2×2 matrix is defined as $\mathbf{H}_m(z)$ with m standing for modulated versions of the filters $H_0(z)$ and $H_1(z)$ contained in this matrix. In this linear time-varying system, there are three drawbacks need to be addressed: aliasing distortion, amplitude distortion, and phase distortion [12]. First of all, in order to avoid the aliasing distortion, $X(-z)$ which represents the aliased version of the input signal should be eliminated, and this leads to

$$[G_0(z)\quad G_1(z)] = C(z)[H_1(-z)\quad -H_0(-z)] \tag{1}$$

where $C(z)$ is a nonzero polynomial [24]. Then the system is turned