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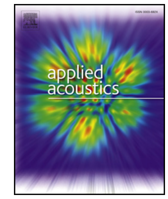

Vibro-acoustic properties of sandwich structures

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ABSTRACT

The vibro-acoustic properties of three-layered sandwich elements depend on the bending stiffness of the structure. It is found that the apparent bending stiffness of a sandwich beam depends on frequency, its length and boundary conditions. A beam with free ends appears stiffer than the same beam clamped at both ends. The apparent bending stiffness of a sandwich beam with clamped ends decreases as the length of the beam is reduced. The opposite is the case for a beam with free ends. A method for estimating the material parameters of a sandwich beam is described. The frequency and space averages of the point mobility of a finite sandwich beam is found to be equal to the point mobility of an infinite sandwich beam of the same type. Coupling loss factors between sandwich beam elements for some simple types of junctions are given.

1. Introduction

Sandwich elements, or building blocks, with lightweight cores have been used since the days of the Romans. Sandwich elements are nowadays not only used for buildings but also by the vehicle industry. The term sandwich element refers to a structure with a fairly thick lightweight core with thin laminates bonded to each side of either a foam or honeycomb core. This type of construction combines low weight with high strength. The aim has always been to reduce the weight as much as possible while maintaining the strength of the structure. However, for a vehicle manufacturer not only weight and safety requirements but also comfort criteria for driver and passengers must be considered. A drawback with lightweight and stiff structures is their often poor acoustic properties. Thus the added challenge is to optimize the acoustic properties of sandwich structures while maintaining low weight and high strength of the structure. Any optimizing scheme requires a good understanding of the vibro-acoustic properties of the material being used.

The sound pressure levels in a traditional vehicle manufactured from metal plates and beam elements can to a certain extent be estimated by means of numerical methods such as FEM, BEM, SEA and various hybrids of these techniques. The numerical methods in combination with measurements on real structures can be used to predict the acoustic quality already at the design phase of a new model. However, for a product completely or partially made of sandwich elements the same prediction tools developed for thin plate elements cannot readily be used. The reason is that the dynamic properties of sandwich elements strongly depend on frequency as demonstrated in [1] through [4]. The vibration of thick structures and sandwich elements are discussed in the Refs. [5–22]. For any prediction of noise levels in a vehicle vibro-acoustic properties like sound transmission loss, sound radiation ratio, energy flow, modal densities and coupling loss factors must be modelled for all construction elements which are part of the entire structure.

Various models for the prediction of the sound transmission loss and the sound radiation ratio for sandwich structures are presented in Refs. [1] and [4], and [10–13]. The influence of boundary conditions and plate dimensions are not considered in these references. In [1] the concept of an apparent frequency dependent bending stiffness is introduced to calculate the sound transmission loss of sandwich structures. Lu and Xin have in [23] presented a number of numerical models for calculating the acoustic properties of sandwich structures.

2. Sandwich elements with foam or honeycomb cores

In general sandwich or honeycomb constructions are symmetric with respect to the centreline. The Young's modulus for a laminate is high and much higher than the corresponding modulus for the core. A laminate can be treated as thin satisfying Kirchhoff plate or Bernoulli-Euler beam models as long as the wavelength for pure bending waves propagating in the structure is larger than six times the thickness of the plate as suggested by Heckl [24].

A honeycomb core can be modelled as an equivalent foam core [4]. In general both the in-phase and anti-phase motion of the laminates must be considered. However, for typical foam and honeycomb cores, the stiffness of the core material in the direction perpendicular to the

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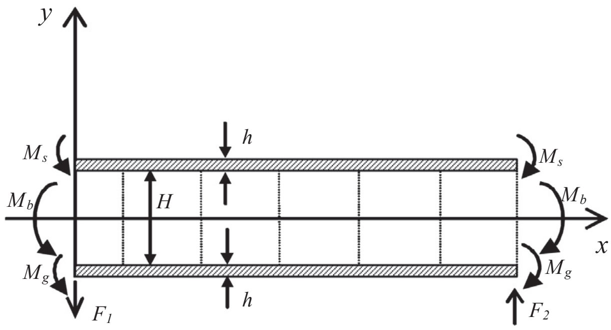

Fig. 1. Excitation of a sandwich beam by an external force and resulting forces and moments.

laminates is sufficiently high to allow only the in-phase motion of the laminates except in the very high frequency range.

The lateral deflection of a sandwich panel is primarily caused by bending but also by shear in and rotation of the core. The total lateral displacement w of a sandwich beam is a result of the angular displacement due to bending of the core as defined by β and the angular displacement γ due to shear in the core. For a beam oriented along the x-axis of a coordinate system $\partial w/\partial x = \gamma + \beta$. The differential equations governing w, β and γ can be determined using Hamilton's principle as discussed in [2,4]. In deriving the equations governing the lateral displacement of the structure shown in Fig. 1 symmetry of the beam is assumed. However, corresponding differential equations can be derived for non-symmetric sandwich beams. The identical laminates shown in Fig. 1 have a Young's modulus E_l, bending stiffness D_2, density ρ_l and thickness h. The effective shear stiffness of the core is G_c, its Young's modulus E_c, its equivalent density ρ_c and its thickness H. The parameter G_c is for a thick beam, plate or core not necessarily equal to the shear stiffness G of the material as suggested by Timoshenko [6]. As compared to the laminates the core itself is assumed to have a very low modulus of elasticity E_{cx} in the x-direction. It is assumed that foam and honeycomb cores can be modelled in the same way by assuming that shear and bending stiffness representing some space averages for the core. In the y-direction, the core is considered to be sufficiently stiff to ensure that the laminates move in phase within the frequency range of interest. The static bending stiffness D_1' and the mass moment of inertia I_ω' of the beam shown in Fig. 1 and having a width b, are

$$D_1' = b[E_{cx}H^3/12 + E_l(H^2h/2 + Hh^2 + 2h^3/3)] \tag{1}$$

$$I_\omega' = b[\rho_c H^3/12 + \rho_l(H^2h/2 + Hh^2 + 2h^3/3)] \tag{2}$$

In general $E_l >> E_{cx}$. The bending stiffness of one laminate is $D_2' = bE_lh^3/12$. The mass per unit length of the entire beam is $m' = \mu b = b(2h\rho_l + H\rho_c)$ where μ is the mass per unit area of the beam. The cross section area of the core is $S = b{\cdot}H$.

The beam in Fig. 1 is assumed to be excited by a force $F'(x,t)$ per unit length. Force and displacement are positive in the direction of the positive y-axis. F_1 and F_2 are the resulting forces at the ends of the beam. The shear deformation of the core induces a bending of each laminate resulting in a bending moment M_s on the laminate and giving an angular displacement γ. The total bending moment M_b acting on the entire beam is caused by pure bending defined by the angular displacement β. As demonstrated in [2,4] the displacement w and the angular displacement β caused by an external force F' per unit length of the beam must satisfy the differential equations

$$-G_cS\left\{\frac{\partial^2 w}{\partial x^2}-\frac{\partial \beta}{\partial x}\right\} + 2D_2'\left\{\frac{\partial^4 w}{\partial x^4}-\frac{\partial^3 \beta}{\partial x^3}\right\} + m'\frac{\partial^2 w}{\partial t^2} - F' = 0 \tag{3}$$

$$-G_cS\left\{\frac{\partial w}{\partial x}-\beta\right\} - D_1'\frac{\partial^2 \beta}{\partial x^2} + 2D_2'\left\{\frac{\partial^3 w}{\partial x^3}-\frac{\partial^2 \beta}{\partial x^2}\right\} + I_\omega'\frac{\partial^2 \beta}{\partial t^2} = 0 \tag{4}$$

Using Eqs. (3) and (4) to eliminate β, the equation governing w is obtained as

$$-2D_1'D_2'\frac{\partial^6 w}{\partial x^6} + 2D_2'I_\omega\frac{\partial^6 w}{\partial x^4\partial t^2} + G_cSD_1'\frac{\partial^4 w}{\partial x^4} - [(D_1' + 2D_2')m' + G_cSI_\omega']\frac{\partial^4 w}{\partial x^2\partial t^2} + G_cSm'\frac{\partial^2 w}{\partial t^2} + m'I_\omega\frac{\partial^4 w}{\partial t^4} = G_cSF' - (D_1' + 2D_2')\frac{\partial^2 F'}{\partial x^2} + I_\omega'\frac{\partial^2 F'}{\partial t^2} \tag{5}$$

Eliminating w instead gives the corresponding equation for β. For no external force on the structure, $F' = 0$, w and β satisfy the same differential equation.

The boundary conditions to be satisfied are also obtained from the variational expression as shown in [2]. The requirements to some elementary boundary conditions for sandwich beams are:

Simply supported edge

$$w = 0;\quad \partial\beta/\partial x = 0;\quad \partial^2 w/\partial x^2 = 0 \tag{6}$$

Clamped edge

$$w = 0;\quad \beta = 0;\quad \partial w/\partial x = 0 \tag{7}$$

Free edge

$$\partial\beta/\partial x = 0;\quad \partial^2 w/\partial x^2 = 0;\quad D_1'\partial^2\beta/\partial x^2 - I_\omega'\partial^2\beta/\partial t^2 = 0 \tag{8}$$

3. Wavenumbers

By setting $w = W{\cdot}\exp[i(\omega t - k_x x)]$ in the wave equation (5) and by assuming the external forces to be zero the dispersion relation is obtained as

$$2D_1'D_2'k_x^6 - 2D_2'I_\omega k_x^4\omega^2 - [m'(D_1' + 2D_2') + I_\omega'G_cS]k_x^2\omega^2 + G_cS[D_1'k_x^4 - m'\omega^2] + m'I_\omega'\omega^4 = 0 \tag{9}$$

There are six solutions to Eq. (9). These solutions are written $k_x = \pm\kappa_1, \pm i\kappa_2, \pm i\kappa_3$ where κ_1 and κ_3 are always real whereas κ_2 shifts from being real in the low frequency region to being imaginary for high frequencies. For one particular structure the wavenumbers or rather their absolute values are shown in Fig. 2. The material and geometrical parameters for the infinite beam are: $h = 1$ mm; $H = 10$ mm; $E_l = 70{\cdot}10^9$ Pa, $E_c = 0.13{\cdot}10^9$ Pa, $G_c = 45{\cdot}10^6$ Pa, $\rho_l = 2700$ kg/m^3, $\rho_c = 74$ kg/m^3. The parallel lines in the figure define the upper and lower asymptotes of the wavenumbers describing propagating waves. The bottom straight line represents the wavenumber for flexural waves propagating in a slender or Bernoulli-Euler beam with a static bending

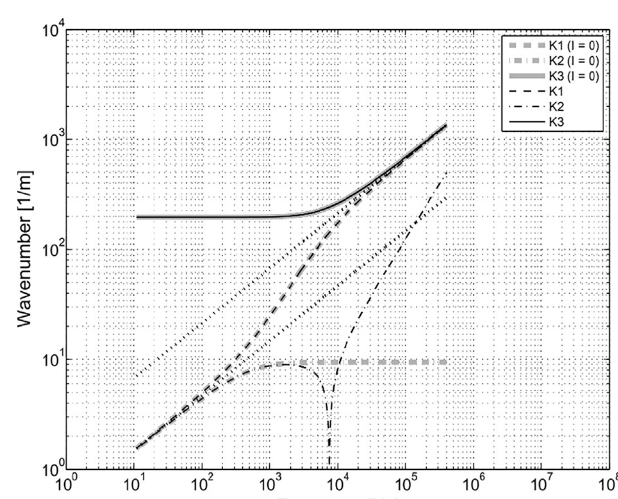

Fig. 2. Wavenumbers for the flexural motion of a sandwich beam calculated for two cases; 1. The mass moment of inertia of the beam included. 2. The mass moment of inertia set to equal zero. The straight dotted lines are the low and high frequency asymptotes for the propagating waves.