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Wave based method for vibration analysis of double-walled cylindrical shells

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ABSTRACT

Wave based method (WBM) is presented for vibration analysis of double-walled cylindrical shells coupled by annular plates. To establish the governing equation, the double-walled shell is divided into several cylindrical shell segments and annular plates according to junctions of discontinuities, and they can be easily synthesized through continuity conditions at the junctions. Flügge shell theory and thin plate theory are utilized to describe motions of cylindrical segments and annular plates, displacements of which are expanded as wave functions. In addition, artificial springs are employed to restrain displacements at boundaries to uniformly consider elastic and classic boundary conditions. To test the validity of WBM, free and forced vibration results of WBM are compared with those of literature and finite element method, which demonstrates high accuracy of present method. Some mode shapes are also given for visual understanding. Furthermore, effects of elastic boundary conditions and annular plates on free and forced vibrations are investigated. Results show that axial and circumferential displacements are of great effect on fundamental and beam mode frequencies, and effects of the number of annular plates are obviously greater than the thickness of annular plates. The results of present paper can be used as benchmarks for further researches.

1. Introduction

Double-walled cylindrical shells coupled by annular plates are extensively used in submarines, coolers, evaporators, multistage pumps and so forth. In practical engineering applications, the double-walled shells are subjected to various dynamic loads which can cause excessive vibration, noise and fatigue damage, and understanding accurately vibration characteristics of the structures is the most efficient means to avoid or reduce those negative effects in design process. Some researchers have investigated vibration characteristics of those structures. However, due to the complexity involved in modeling and solution, the literature is very little. Although finite element computer programs like ANSYS, NASTRAN and ABAQUS have been well developed and can be utilized to analyze vibrations of these complex shells, FEM has obvious disadvantages in computational efficiency. For example, researchers or engineers have to classify all modes one by one to identify the mode shape of a certain frequency, which is a time-consuming and tiresome process. Furthermore, to meet the requirements of convergence, a huge number of elements are needed for high frequencies, which significantly increases the computation time and storage. In this context, proposing an accurate and efficient method to analyze vibrations of the double-walled cylindrical shells is considerable and meaningful.

Although two coaxial cylindrical shells are generally coupled by fluid [1–9], poroelastic material [10–13] or other similar materials, annular plates are also widely used to couple two coaxial cylindrical shells, and some scholars have studied static and dynamic characteristics of these plate-cylindrical structures. Smith and Haft [14] presented an exact solution for free vibrations of a cylindrical shell clamped at one end and closed by an elastic circular plate at the other end. Irie et al. [15] and Yamada et al. [16] adopted the transfer matrix method to calculate natural frequencies and mode shapes of a single cylindrical shell with annular plates at two ends and a concentric double-walled cylindrical shell closed by annular plates at two ends, respectively. Tavakoli and Singh [17] used the state space method to study vibrations of a cylindrical shell closed by two circular plates. Cheng and Nicolas [18] used the variational principle to analyze free vibrations of a finite circular cylindrical shell with various boundary conditions at one end and a circular plate at the other end. Huang and Soedel [19] and Yim et al. [20] adopted the receptance method to analyze modal characteristics of a simply supported and clamped-free cylindrical shell with a circular plate attached at arbitrary axial position, respectively. On basis of the Rayleith-Ritz approach, Yuan and Dickinson [21] utilized the concept of artificial springs to assemble cylindrical shells and circular/annular plates together, and natural frequencies and mode shapes of a double-walled cylindrical shell with two annular plates at each end were analyzed. Tso and Hansen [22] adopted the wave propagation approach to study the transmission of vibration waves through the junctions of an annular plate and an

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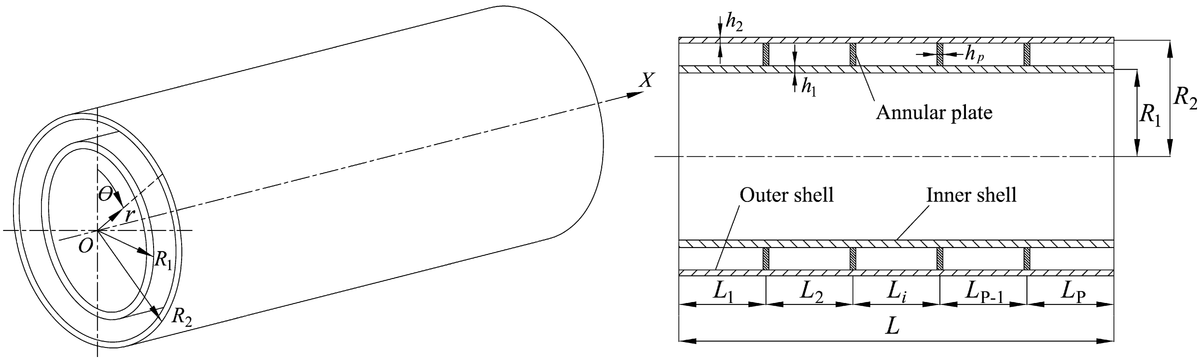

Fig. 1. Schematic diagram of double-walled cylindrical shell.

infinite cylindrical shell. B-spline finite elements were employed by Benjeddou [23] to study free vibration of a cylindrical shell with two circular end plates. Sergei et al. [24] studies low frequencies and vibration modes of a thin cylindrical shell joined with an annular plate by means of the asymptotic and numerical methods. Ding [25,26] put forward a matrix analysis method to study strength and stability of annular plate-coupled double cylindrical shells subjected to hydrostatic external pressure. Bai [27] adopted the modal superposition method to analyze the sound and vibration transmission characteristics of simply supported double-walled cylindrical shells with continuous or non-continuous annular plates. Gin et al. [28] developed a modified Fourier series solution to analyze vibrations of conical shells with general boundary conditions, and then the method was extended to study vibrations of coupled plate-cylindrical shell structures, including circular cylindrical double-shell structures [29] and elastically coupled annular plate and cylindrical shell structures [30]. The research group of the authors used annular plates to describe motions of ring-stiffeners with rectangular cross-sections and the high accuracy was demonstrated [31–33].

From the literature reviews above it can be clearly observed that, although there are many papers investigating annular plate-cylindrical shell structures, only Refs.[16,21,25–27,29] took into account double-walled cylindrical shells. However, annular plates were just located at the ends and only free vibrations were discussed in Refs.[16,21]. In Refs.[25,26], static characteristics, rather than vibration characteristics, were analyzed. In addition, except for Ref.[29], only classic boundary conditions, such as clamped (C), simply supported (S) and free (F) ones, were considered in those cited literature. In practice, much more complex boundary conditions need to be analyzed. To this end, developing a method to deal with arbitrary boundaries, including classic and elastic ones, is also meaningful.

The main purpose of this paper is to present an accurate and efficient method to analyze both free and forced vibrations of double-walled cylindrical shells with arbitrary distributed annular plates and arbitrary boundary conditions. WBM, which has been utilized by the authors in Refs.[31,32,34], is extended to establish vibration governing equation of double-walled cylindrical shells with annular plates. The double-walled shell is firstly divided into several substructures according to the locations of annular plates. Then, displacement functions of homogeneous cylindrical segments and annular plates are expanded as wave functions, rather than general polynomials, trigonometric functions or Fourier series, which is the essence of WBM. As a result, four displacements and four forces at boundaries of cylindrical shells/annular plates can be expressed in terms of eight wave contribution coefficients. Four artificial springs with appropriate stiffness constants are used to restrain four displacements at edges. Finally, all substructures are assembled by using continuity conditions of adjacent substructures, and governing equation is correspondently established with the help of boundary conditions. It should be mentioned that, although the present method (WBM) is similar with the dynamic stiffness method (DSM) and some other methods, the essence of WBM and DSM is different. Dynamic stiffness matrices relating displacements and forces at the two ends of substructures must be deduced in DSM whereas they are not required in WBM and continuity conditions of adjacent substructures are directly used to establish the governing equation, so the governing equation of WBM is more exhibitory. On the other hand, based on the displacement functions of WBM, dynamic stiffness matrices of substructures can be deduced so that WBM can be easily extended to DSM. In addition, WBM is believed to include following novelties. First, it models double-walled cylindrical shells coupled by arbitrarily distributed annular plates, the literature about which is rare. Second, it offers an accurate and efficient analytic method to investigate both free and forced vibrations of the double-walled shells. Finally, it is applicable to deal with both classical and elastic boundary conditions.

2. Governing equation

Fig. 1 shows the schematic diagram of a double-walled cylindrical shell with uniformly distributed P−1 annular plates. Mean radii of inner and outer shells are R_1 and R_2, and corresponding thicknesses are h_1 and h_2. h_p is thickness of annular plates and L is axial length. In the figure, the global cylindrical coordinate system, (r,θ,X), is also presented.

To establish governing equation of free and forced vibrations, the double-walled shell is divided at locations of annular plates. After division, there are $2P$ cylindrical segments, including inner and outer cylindrical segments, and P−1 annular plates to be analyzed.

2.1. Cylindrical shell segments

Fig. 2 shows the local cylindrical coordinate system (r,θ,x) of a cylindrical segment. In the meantime, positive directions of four displacement resultants and four force resultants are given.

Based on Flügge shell theory, motion equations of cylindrical shells are [35]

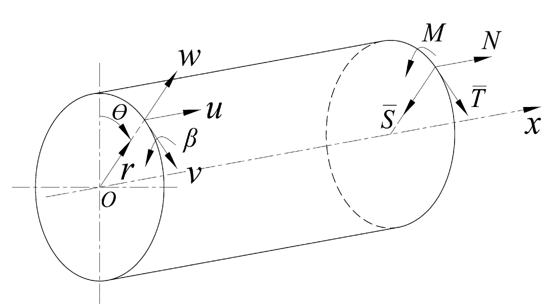

Fig. 2. Local coordinate system, positive directions of displacements and forces of a cylindrical segment.