Applied Acoustics 137 (2018) 18–26

Contents lists available at ScienceDirect

Applied Acoustics

journal homepage: www.elsevier.com/locate/apacoust

A stochastic perturbation finite element-least square point interpolation method for the analysis of uncertain structural-acoustics problems with random variables

G.Q. Jiang, L.Y. Yao*, F. Wu

College of Engineering and Technology, Southwest University, Chongqing 400715, PR China

ARTICLE INFO

Keywords:
Stochastic analysis
SP-FEM
Monte Carlo
SP-FE-LSPIM
Structural-acoustics problems

ABSTRACT

In this paper, a novel stochastic perturbation finite element-least square point interpolation method (SP-FE-LSPIM) is introduced to improve the calculation accuracy for analyzing structural-acoustics problems. Inherited the element-compatibility of finite element method and the quadratic polynomial completeness of LSPIM, the present method obtains the global shape function for partition of unity (PU) and the least square point interpolation for local approximation. Besides, a first-order perturbation technique is also introduced into this theory for probabilistic analyzing. Thus, the response of the coupled systems can be expressed simply as a linear function of all pre-defined input variables by using the change-of-variable techniques. Due to the linear relationships between variables and the response, the probability density function and the cumulative probability density function of response can be obtained based on a simple mathematical transformation of probability theory. So the proposed approach not only improves the numerical accuracy of deterministic output quantities with respect to a given random variable, but also handles the randomness well in the systems. One numerical example for frequency response analysis of random structural-acoustics is presented and verified by Monte Carlo (MC) simulation and stochastic perturbation finite element method (SP-FEM) to demonstrate the effectiveness of the present method.

1. Introduction

When a dynamic load is applied, engineering systems such as vehicles, sea vessels, and air crafts encounter structural-acoustics problems, which range mostly from the “low-frequency” to “high-frequency” regime. In order to cover different frequency ranges, different types of numerical simulation techniques have been developed for the structural-acoustic predictions. Most can be categorized into two major groups: deterministic and statistical. The former is suitable for the lower frequency range, while the latter is suitable for the higher.

Finite element method (FEM) [1–4], a typical deterministic method, is successfully used to model the low frequency vibrational and acoustical behaviors of structural-acoustic systems. However, it will suffer from numerical dispersion when solving the acoustic wave equation, which means that the phase error of numerical wave will disperse significantly for high wave number problems. There are two ways to obtain reliable results in solving the high wave number problems: discretizing the problem domain into smaller and more accurate elements and using higher order polynomial approximation function, but these will lead to the increase of computing time and memory space. The meshfree method [5–8], with the characteristic of high precision and with not needing to divide the grid, can effectively avoid some disadvantages of FEM. Nevertheless, it cannot impose boundary conditions directly because of lack of the Kronecker properties, which results in a decrease in computational efficiency. Melenk et al. [9,10] proposed a mixed finite element-meshfree method to analyze various mechanical problems by combining FEM and mesh-free technique. The basic principle is to construct high order global finite element formula by increasing the order of local support function without increasing the support point. Cui and co-workers proposed some novel numerical methods to significantly improve the computation accuracy for solving acoustic problems [11–14]. Zhang et al. [15,16] proposed a finite element-least square point interpolation method (FE-LSPIM) to effectively analyze the statics and dynamics. Later, Yao et al. [17] obtained good results by using FE-LSPIM for analyzing the two-dimension acoustic.

Recently, the focus of exploiting a suitable prediction technique for structural-acoustic problems has shifted to the mid-frequency regime, in which the critical factor is uncertainty. Owing to the difficulties involved in developing a reliable deterministic method for the prediction of higher frequency domain, researchers now focus on developing

* Corresponding author.
E-mail address: 19831022y@163.com (L.Y. Yao).

https://doi.org/10.1016/j.apacoust.2018.03.003
Received 10 March 2017; Received in revised form 6 December 2017; Accepted 4 March 2018
Available online 09 March 2018

efficient methods that take into account model uncertainties and variability [18]. Probabilistic methods are mostly chosen with acceptable efficiency to handle the uncertainties, which are expressed by using different types of random variables [19]. The Monte Carlo simulation method (MCM) [20–23], the most robust and simplest probabilistic method, can always offer the most direct and reliable results at the expense of the large computational load. Therefore, the MCM results are chosen as the reference results in this work. The spectral stochastic finite element method (SSFEM) [24–26] and stochastic perturbation finite element method approach (SPFEM) [27–30] are another two classical probabilistic methods. In SSFEM, several random Hermite polynomials are used to approximate the response quantity of the systems [26]. In SPFEM, the response vectors or matrixes related to the pre-defined random input variables must be expanded at the first order or higher order approximations according to the Taylor series. Cui's group successfully applied this technique into acoustic problems [27–30]. In this work, the stochastic perturbation technique is introduced into FE-LSPIM.

Like SPFEM, the response vectors or matrixes related to the pre-defined random input variables are expanded based on the Taylor series expansion in hybrid stochastic perturbation FE-LSIPM (SP-FE-LSIPM). For a low variability level of the design parameters, the result will still be accurate if only the first order expansion is applied [31,32]. As the first order approximations are applied, the acoustic pressure response can be approximated as a linear function of the random variable. Thus the computational procedure can be largely simplified, and the expectations and the standard deviations of the response can be easily obtained through the SP-FE-LSIPM. Moreover, a change-of-variable technique [33–37] is introduced to observe the distribution of random responses. There are two significant steps for the calculation of probability density functions (PDF) and cumulative density functions (CDF): firstly, the first order Taylor expansion technique produce the linear functions between the inputs and responses; secondly, the change-of-variable technique can help to produce the PDF and CDF of responses. Thus the promising approach for the comprehensive analysis of uncertainties can be acquired by combining SP-FE-LSPIM with the change-of-variable technique.

Considering the superior performance of FE-LSPIM for the structural-acoustic problem, one is quite sure that the change-of-variable technique can be introduced into the FE-LSPIM frame and a hybrid stochastic perturbation FE-LSPIM frame (SP-FE-LSPIM) can be presented as a promising method to solve the structural-acoustic problem. The newly constructed SP-FE-LSPIM will be expected to efficiently and effectively evaluate the response probability density function and cumulative density functions.

2. Structural-acoustic coupling system

As shown in Fig. 1, the structural-acoustics system is composed of the structural domain $\mathbf{\Omega}_s$, the acoustic domain $\mathbf{\Omega}_f$ and the coupling interface $\partial\mathbf{\Omega}_{sf}$. The boundary of structural domain $\mathbf{\Omega}_s$ includes the essential boundary $\mathbf{\Gamma}_u$ and the natural boundary $\mathbf{\Gamma}_t$. The boundary of acoustic domain $\mathbf{\Omega}_f$ includes the structural domain $\mathbf{\Omega}_s$ and a rigid boundary.

2.1. FE-LSPIM formulations for shell structural

In this section, the FE-LSPIM formulations for the shell structural are briefly introduced.

2.1.1. Finite element-least square point interpolation of shell structural

According to the First shear deformation theory shown in Fig. 2, the displacement components u, v and w of the shell structural can be written as below:

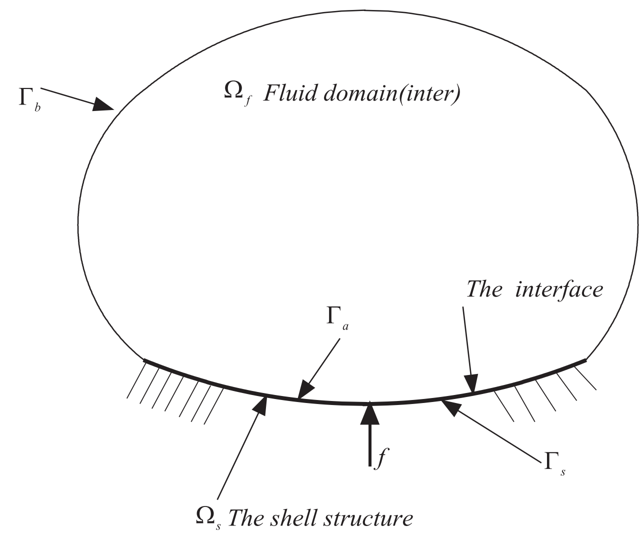

Fig. 1. The structural-acoustic coupling system.

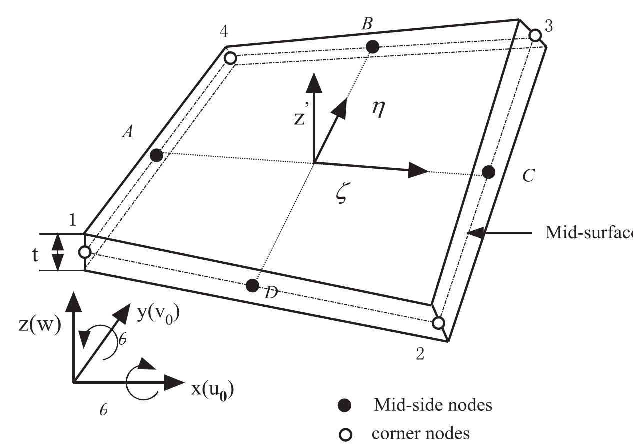

Fig. 2. Schematic of shell element.

$$\begin{cases} u(x,y,z) = u_0(x,y) + z\theta_x(x,y) \\ v(x,y,z) = v_0(x,y) + z\theta_y(x,y), \\ w(x,y,z) = w_0(x,y) \end{cases} \tag{1}$$

where u_0, v_0 and w_0 denote the displacements of three direction x, y and z of the middle surface of shell, respectively. θ_x and θ_y are the rotation angles of XY and YZ surface, respectively.

The strain vector ε can be written as:

$$\varepsilon = [\varepsilon_{xx},\varepsilon_{yy},\gamma_{xy},\gamma_{xz},\gamma_{yz}]^T = \begin{bmatrix} \varepsilon_m \\ 0 \end{bmatrix} + \begin{bmatrix} z\varepsilon_b \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ \varepsilon_s \end{bmatrix} \tag{2}$$

$$\begin{cases} \varepsilon_m = [\partial u_0/\partial x \quad \partial v_0/\partial y \quad \partial u_0/\partial y + \partial v_0/\partial x]^T \\ \varepsilon_b = [\partial\theta_0/\partial x \quad \partial\theta_0/\partial y \quad \partial\theta_0/\partial y - \partial\theta_0/\partial x]^T \\ \varepsilon_s = [\partial w_0/\partial x + \theta_x \quad \partial w_0/\partial y + \theta_y]^T \end{cases}, \tag{3}$$

where ε_m is the membrane stress vector, ε_b is the bending stress vector, and ε_s is the shear stress vector.

The bending stiffness constitutive matrix D_b, the transverse shear stiffness constitutive matrix D_s, and the membrane stiffness constitutive matrix D_m can be expressed as: