Applied Acoustics 136 (2018) 29–35

Contents lists available at ScienceDirect

Applied Acoustics

journal homepage: www.elsevier.com/locate/apacoust

Modal analysis using fiber Bragg gratings

Alec Hammond*, LeGrand Shumway, Christina Hurlbut, Frederick Seng, Ivann Velasco, Jonathan Blotter, Stephen Schultz

Brigham Young University, Provo, UT, United States

ABSTRACT

A new, low-cost modal analysis method, capable of extracting the structural modes, verifying numeric simulations, and fine-tuning musical instruments is presented. This method uses fiber Bragg gratings and is demonstrated by reconstructing the mode shapes of a marimba bar. To verify the practicality and accuracy of this method, the results are compared to a numerical FEA simulation and experimental results using a scanning laser Doppler vibrometer.

1. Introduction

Many strive to simplify and improve the tuning process for percussion instruments [1]. Since the timbre, or quality of sound of an instrument depends on the induced vibrational modes, modal analysis tools are often used. Simulations explore the mechanical properties of the instrument and determine what shape and material will produce the most harmonic overtones [2,3]. The Finite Element Method (FEM), for example, can take even the most complicated instrument designs and analyze their frequency response [4]. Unfortunately, fabrication is never perfect, and manufacturers rely on modal analysis tools to fine-tune the instruments and to match the simulations.

Holographic interferometry and sinusoidal excitation, for example, were developed to simulate and analyze modal patterns [5]. Other techniques such as a 3D scanning laser Doppler vibrometer (SLDV), time averaged holographic interferometry, and speckle-pattern interferometry can be used to extract the modes [6–8]. These techniques provide very high spatial resolution and highly accurate measurements. However, they require very expensive equipment, more test preparation, and a direct line of sight to the structure often making in situ measurements very difficult.

Alternate approaches to tune and analyze the structural and acoustic modes use electromechanical sensors such as accelerometers and microphones. An accelerometer may be attached to the musical instrument and used to extract the structural modes. Although this approach does not require a direct line of sight to the instrument, it does require intrusive physical attachment, provides very low spatial resolution, and depending on placement can miss important mode shapes. The accelerometer also mass loads the instrument shifting the resonance frequencies and distorting the mode shapes [9]. Microphones are an effective approach to measure the radiated acoustic frequencies which can be used for tuning but they do not describe the mode shapes and they often require acoustic chambers such as anechoic and reverberation chambers to make measurements that are not affected by the environment. In this work, we developed and validated a new method to overcome the deficiencies of these other methods. The method is based on fiber Bragg gratings (FBGs) as the measurement device. The method is very low cost, can be setup up quickly, can provide high-spatial resolution, and is significantly less intrusive to the test structure than some methods. An experiment was performed where a musician played a marimba bar with FBGs attached to the bar. With the information gathered by these FBGs, the deflection shape of the marimba bar after each strike was reconstructed as a function of time. These data were used to analyze both longitudinal and torsional modes.

To verify the results of this new FBG method, numerical results from a Finite Element Analysis (FEA) and experimental results from a SLDV were used to compare to the FBG method results.

In the remainder of this paper, the process to extract the mode shapes as a function of time from the FBG method is presented. Next the results from the FEA simulation and the scanning laser system are presented. Finally, the experiment involving the FBGs is described and the results are presented. These experiments were done using a marimba bar due to the wealth of literature available regarding its behavior.

2. System overview

The shape of the marimba bar is measured as a function of time by measuring the torsion and curvature across the bar as a function of time.

* Corresponding author.
E-mail address: alec.hammond@byu.edu (A. Hammond).

https://doi.org/10.1016/j.apacoust.2018.02.004
Received 23 August 2017; Received in revised form 17 December 2017; Accepted 7 February 2018
0003-682X/ © 2018 Elsevier Ltd. All rights reserved.

The FBGs have a cross-sectional diameter of around 0.2 mm and a mass of 0.6 mg resulting in negligible mass loading of the test structure. In addition, multiple sensing elements can be multiplexed serially, allowing several sensors to run on the same signal line. The following section describes how to extract the shape from curvature and torsion using the Frenet-Serret equations. The curvature and torsion extraction process is then outlined. Finally, the strain extraction process and interrogation system are explained.

2.1. Shape extraction

The shape of the marimba bar can be modeled by its neutral axis, a space curve running through its center. Excitation induces various curvature and torsion modes that bend and twist the neutral axis of the marimba bar respectively [10]. Analytical expressions, known as the curvature κ, and torsion functions, model these modes along the length of the bar. Since differential geometry can relate these curvature and torsion functions to the neutral axis of the marimba bar, they can be extracted from the neutral axis's position function and vice versa. The forward process is first explained for clarity. Then, the reverse process capable of extracting the marimba bar's shape from its curvature and torsion measurements is presented.

The neutral axis as a space curve $\vec{r}(s)$is described by

$$\vec{r}(s) = \hat{x}x(s) + \hat{y}y(s) + \hat{z}z(s), \tag{1}$$

where $\hat{x}$,$\hat{y}$,$\hat{z}$are orthonormal unit vectors and $x(s)$,$y(s)$, and $z(s)$ are their corresponding functions parametrized by arc length.

The Frenet-Serret equations relate the curvature and torsion functions to the marimba bar's neutral axis. First, a Frenet Frame consisting of the marimba bar's tangent $\vec{T}(s)$, normal $\vec{N}(s)$, and binormal $\vec{B}(s)$ vector functions is established as given by

$$\vec{T}(s) = \frac{\frac{d}{ds}\vec{r}(s)}{\|\frac{d}{ds}\vec{r}(s)\|} = \frac{\hat{x}\frac{d}{ds}x(s) + \hat{y}\frac{d}{ds}y(s) + \hat{z}\frac{d}{ds}z(s)}{\sqrt{\left(\frac{d}{ds}x(s)\right)^2 + \left(\frac{d}{ds}y(s)\right)^2 + \left(\frac{d}{ds}z(s)\right)^2}} \tag{2}$$

$$\vec{N}(s) = \frac{\frac{d^2}{ds^2}\vec{r}(s)}{\|\frac{d^2}{ds^2}\vec{r}(s)\|} = \frac{\hat{x}\frac{d^2}{ds^2}x(s) + \hat{y}\frac{d^2}{ds^2}y(s) + \hat{z}\frac{d^2}{ds^2}z(s)}{\sqrt{(\frac{d^2}{ds^2}x(s))^2 + \left(\frac{d^2}{ds^2}y(s)\right)^2 + \left(\frac{d^2}{ds^2}z(s)\right)^2}} \tag{3}$$

$$\vec{B}(s) = \vec{T}(s) \times \vec{N}(s) \tag{4}$$

The Frenet-Serret equations, which relate the tangent $\vec{T}$, normal $\vec{N}$, and binormal $\vec{B}$ vectors to the corresponding curvature κ and torsion τ functions are given by

$$\frac{d}{ds}\begin{bmatrix}\vec{T}(s)\\ \vec{N}(s)\\ \vec{B}(s)\end{bmatrix} = \begin{bmatrix}0 & \kappa(s) & 0\\ \kappa(s) & 0 & \tau(s)\\ 0 & \tau(s) & 0\end{bmatrix}\begin{bmatrix}\vec{T}(s)\\ \vec{N}(s)\\ \vec{B}(s)\end{bmatrix} \tag{5}$$

Using the curvature κ and torsion τ functions along with initial conditions $\vec{T}(0)$,$\vec{N}(0)$, and $\vec{B}(0)$, the vectors $\vec{T}$,$\vec{N}$, and $\vec{B}$, can be solved numerically using any ODE solver. Once functions for $\vec{T}$,$\vec{N}$, and $\vec{B}$ are found, the position function $\vec{r}(s)$ is found by numerically integrating $\vec{T}(s)$, as given by

$$\vec{r}(s) = \int \vec{T}(s)ds + \vec{r}(0) \tag{6}$$

If only one neutral axis is defined for the entire system (as is the case with the marimba bar) then the constant$\vec{r}(0)$ is arbitrary and can be ignored. Once the neutral axis is reconstructed, a rotational minimizing frame is constructed to simulate the marimba bar as a rectangular prism, instead of a simple line. More information regarding the shape extraction process can be found in [11].

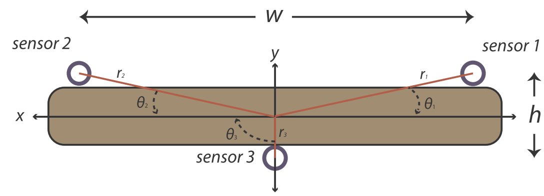

Fig. 1. A cross-section of the marimba bar at a sensor triplet. The neutral axis runs through the center of the marimba along the z-axis.

2.2. Curvature and torsion extraction

In order to extract the curvature and torsion functions from the optical strain sensor data, the sensors must be strategically placed to reflect the neutral axis of the marimba bar. With the geometric configuration found in Fig. 1, the neutral axis is defined along the center of the marimba bar along the z-axis (coming out of the plane). The distances from the center of the marimba bar to sensors 1, 2, and 3 are r_1,r_2, and r_3 respectively. The angle between sensors 1, 2, and 3 and the neutral axis are θ_1,θ_2, and θ_3 respectively. The distance h represents the vertical distance between sensors 1 or 2 and sensor 3, and is essentially the thickness of the marimba bar. The horizontal distance between sensors 1 and 2 (essentially the width of the marimba bar) is defined as w.

A complex curvature vector at that point along the marimba bar is calculated as given by

$$\vec{\kappa} = -\sum_{i=1}^{N} \frac{\epsilon_i}{r_i} e^{j\theta_i} \tag{7}$$

where N is the number of fibers (3 in this case), ϵ_i is the measured strain on the ith fiber, r_i is the distance from the neutral axis, and j is the imaginary unit $\sqrt{-1}$. The real and imaginary parts of the complex curvature vector represent the x and y components of the net strain field, which correspond to the transverse and longitudinal modes respectively. The magnitude and phase of the complex curvature vector represent the curvature and bend angle at that point along the marimba bar.

Multiplexing multiple sensor triplets provides more curvature and torsion data points along the length of the marimba bar. Fig. 2 illustrates three discrete "triplets", where each triple combination provides one curvature vector.

Several curve fitting techniques capable of interpolating the transverse and longitudinal strains along the length of the marimba bar are available [12]. In this work, a local SDOF method is used, where a Discrete Fourier Transform is taken of both the transverse and longitudinal directions for each grating triplet. Other spectral estimation techniques, like tri spectrum averaging [12], are often used to extract a more accurate representation of the frequency response. The frequency

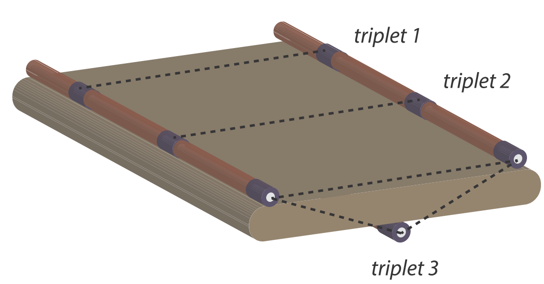

Fig. 2. Three sensor triplets that each provide one curvature and bend angle data point.

Download English Version:

https://daneshyari.com/en/article/7152227

Download Persian Version:

https://daneshyari.com/article/7152227

Daneshyari.com