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Point excitation of a coupled structural-acoustical tire model with experimental verification: Higher order cavity modes

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ABSTRACT

The acoustical modes that exist within a tire's interior cavity have an important influence on tire-road noise, particularly structure-borne noise. The first circumferential mode is most commonly considered: it occurs when the average circumference of the tire interior cavity is approximately one wavelength. However, that mode is just the first of a family of circumferential and radial modes, the latter of which, in particular, are rarely considered, but which were the primary focus here. A modal expansion method was utilized to solve for the point excitation of a fully-coupled, ring-like structural-acoustical tire model. From that model, the dispersion features associated with the radial acoustical modes, which typically first appear between 1400 and 2000 Hz depending on the tire geometry, were identified, and their existence was verified by making detailed measurements on tires. The surface radial velocity of the tire was measured using a laser Doppler velocimeter (LDV), and a point-by-point measurement method was proposed to minimize the measurement error that can occur due to the tangential motion of the treadband.

1. Introduction

The acoustical modes in a tire's interior cavity were identified nearly three decades ago, and they have a strong influence on tire road noise, especially structure-borne noise [1–3]. Moreover, their dynamic influence can also be detected in the surface vibration of the tire [4]. The most prominent air cavity resonance occurs when the acoustic wavelength is approximately equal to the average tire circumference. The natural frequency of that mode in an automobile tire usually ranges from 200 Hz to 230 Hz depending on the size of the tire, and that mode is capable of creating strong structure-borne sound transmission into the vehicle interior, through the rim, hub and suspension, due to the net force the mode produces on the rim. Higher order resonances of this acoustic wave in the circumferential direction occur at approximately integer multiples of the first resonance frequency, but with generally lower strength.

Bolton et al. [5] applied a wavenumber decomposition approach, which used position data and the measured frequency response functions of a tire's surface transfer mobility, to experimentally identify the treadband wave dispersion relations in a typical tire structure. In addition, multiple analytical and finite element tire models have also been formulated to help investigate tire noise generation and radiation problems. For example, Kropp [6] used a two-dimensional circular ring model on an elastic foundation to simulate the vibration behavior of a static unloaded tire, and concluded that the tire treadband vibration was mostly tension controlled at low frequency, and behaved more like a beam/plate in the higher frequency range. Pinnington [7] applied a more comprehensive curved beam model to represent the tire, which included bending stiffness, longitudinal stiffness and shear stiffness. The model also accounted for factors like tension, pressure, speed, etc. Realistic mobility results were obtained from the analytical model, while dispersion relations up to 4 kHz were analyzed and explained. More recently, Kindt et al. [8] developed a two-dimensional ring model with the addition of axial motion represented as a single degree freedom spring-mass system, thus allowing for axial motion of the treadband. They were able to reproduce measured mobility results, and further verified the analytical model by using an equivalent finite element model up to 300 Hz. Molisani et al. [9] had earlier developed a closed form analytical tire model using Donnell–Mushtari shell theory to model the tire tread, and a rigid inner shell to model the wheel rim in order to study the vibration of tire due to the cavity resonances.

Choi and Bolton [10] subsequently used a finite element model to simulate the dynamic characteristics of a pressurized tire and showed that it was possible to observe the influence of acoustic waves within the tire cavity on the tire surface vibration. Note that the latter is possible since the sound field within the tire's internal air cavity connects with the tire radially (i.e., acoustic waves in the tire cavity propagate in both radial and circumferential directions). That is, rather

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than acting simply as a mass loading while simultaneously creating a static pressure in the radial direction, the air cavity also interacts dynamically with the tire in the fully-coupled case. Thus, the sound pressure within the air cavity affects the vibration of the tire, while that vibration, in turn, influences the sound pressure distribution within the air cavity.

In this paper, a damped, fully-coupled tire model with point excitation is described so that the influence of the higher order radial acoustical modes can be demonstrated analytically. In conjunction with the analytical model, experimental results measured using a passenger car tire are presented for comparison. In particular, as previously predicted, by using a free vibration tire model [11], the contribution of radial acoustical modes in the tire cavity to the tire surface vibration have been measured for the first time, and it is shown that their dispersion characteristic can be accurately predicted by the analytical model. Finally, some aspects of the scanning laser measurement technique and associated measurement error are illustrated qualitatively by using the fully-coupled model.

2. Model description

The coupled structural-acoustical tire model is shown in Fig. 1. It is a two-dimensional model, consisting of a circular rigid wheel, an annular air cavity and a flexible circular tire tread. The model was assumed to be stationary (i.e., non-rotating) for all the analysis presented this paper. We have also simplified what is a three-dimensional problem, in the practical case, to an in-plane two-dimensional problem by assuming the tire has no width, thus assuming that there is no motion of the treadband in the axial direction. This simplification means that both the cross-sectional flexural modes and the shell axial dynamics [12] of the treadband are neglected, while the sidewall radial and tangential stiffness effects are preserved. That is, in this work, the sidewalls were replaced by distributed radial and tangential springs having constant stiffnesses. As for the external point loads, both radial and tangential loads were applied at the same, arbitrarily-selected point: the ratio between the loads could be adjusted to represent a single load pointing in an arbitrary direction. Finally, the zero-width assumption also means that the axial acoustic modes of the air cavity have been neglected.

2.1. Equations of motion for the ring structure

The equations of motion for the coupled tire model were derived based on a ring model coupled with a circular air cavity. As described in Soedel's work [13], the ring structure is assumed to be thin and to conform to Love's assumptions. Deformation is assumed small, shear strains are assumed to be zero, rotatory inertia is ignored and the mid-surface of the ring remains the mid-surface after bending. Damping effects were considered here by introducing a complex Young's modulus, $E(1 + j\eta)$, with a loss factor, η, which is also referred to as the hysteretic damping factor. The two variables representing the radial and circumferential displacements of the ring are denoted as w and u, respectively.

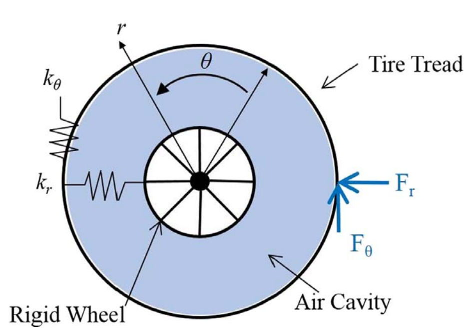

Fig. 1. Analytical model of a coupled tire subject to point excitations.

The equations of motion for the ring structure with internal static pressure loading and distributed stiffness is given by Huang and Su [14] as

$$\frac{Eh^3}{12R^4}\left(\frac{\partial^3 u_r}{\partial\theta^3}-\frac{\partial^2 u_\theta}{\partial\theta^2}\right)-\frac{Eh}{R^2}\left(\frac{\partial u_r}{\partial\theta}+\frac{\partial^2 u_\theta}{\partial\theta^2}\right)-\frac{\sigma_\theta h}{R^2}\left(2\frac{\partial u_r}{\partial\theta}+\frac{\partial^2 u_\theta}{\partial\theta^2}\right) + \left(\frac{p_0}{R}+k_\theta\right)u_\theta+\rho h\frac{\partial^2 u_\theta}{\partial t^2}=q_\theta, \tag{1}$$

$$\frac{Eh^3}{12R^4}\left(\frac{\partial^4 u_r}{\partial\theta^4}-\frac{\partial^3 u_\theta}{\partial\theta^3}\right)+\frac{Eh}{R^2}\left(u_r+\frac{\partial u_\theta}{\partial\theta}\right)+\frac{\sigma_\theta h}{R^2}\left(2\frac{\partial u_\theta}{\partial\theta}-\frac{\partial^2 u_r}{\partial\theta^2}\right) + \left(\frac{p_0}{R}+k_r\right)u_r+\rho h\frac{\partial^2 u_r}{\partial t^2}=q_r. \tag{2}$$

where u_r and u_θ are the radial and tangential displacement responses, R is the mean radius of the tire structure, k_θ and k_r are the circumferential and radial distributed stiffnesses (and their values were chosen according to Yamazaki and Akasaka's research [15] on tire sidewall stiffness), q_θ and q_r are the distributed loads applied to the ring, p_0 is the static inflation pressure in the tire, and σ_θ is the pretension force in the circumferential direction due to p_0. Here, we assume that the air loading is always pointing from the tire axle towards the tire surface in the radial direction. The normal mode shapes assumed for the tire structural displacement, associated with a circumferential wavenumber, n, are

$$u_r(\theta,t)=\sum_{n=0}^{\infty}A_n e^{-jn\theta}e^{j\omega t}, \tag{3}$$

$$u_\theta(\theta,t)=\sum_{n=0}^{\infty}B_n e^{-jn\theta}e^{j\omega t}, \tag{4}$$

where A_n and B_n are modal amplitude coefficients.

Note that the parameter values used in the computations presented in later sections are given in Table 1. The material-related properties other than k_r and k_θ were taken from Kim and Bolton [16], while the geometry-related properties were measured from one of the tires used in the measurements reported below.

2.2. Air cavity expressions

The air cavity is the annular space formed between the rigid wheel and the flexible tire structure. From Morse and Ingard's *Theoretical Acoustics* [17], the differential equation governing wave propagation in cylindrical coordinates can be written as

$$\frac{\partial^2\psi}{\partial r^2}+\frac{1}{r}\frac{\partial\psi}{\partial r}+\frac{1}{r^2}\frac{\partial^2\psi}{\partial\theta^2}=\frac{1}{c_0^2}\frac{\partial^2\psi}{\partial t^2} \tag{5}$$

where ψ is the velocity potential and c_0 is the ambient, adiabatic sound speed in the cavity. The velocity potential is assumed to be of the form

$$\psi(r,\theta,t)=\sum_{n=1}^{\infty}g_n(r)h_n(\theta)w(t) \tag{6}$$

by separation of variables. Due to the circular geometry of the air cavity, we have

Table 1
Material properties of the coupled tire model.

Parameter	Value	Parameter	Value
Young's Modulus E [Pa]	4.8×10^8	Pressure p_0 [pa]	2.06×10^5
Density ρ [kg/m^3]	1200	Inner Radius r_1 [m]	0.205
Thickness h [m]	0.008	Outer Radius r_2 [m]	0.338
Radial stiffness k_r [N/m]	2×10^6	Tangential stiffness k_θ [N/m]	1×10^6
Loss factor η	0.2		

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