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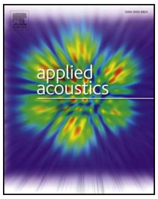

Modeling of duct acoustics in the high frequency range using two-ports

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ABSTRACT

Design of duct networks is challenging because the design should consider the required flow rate, acceptable noise levels, and minimum pressure drop to achieve optimum performance. This paper presents acoustic analysis in high frequency range using sound power two-ports applied to Heating, Ventilation, and Air Conditioning (HVAC) systems. To simulate the acoustic behaviour one need to model three mechanisms; the sound power generated from sound sources (e.g. Fans), the regenerated sound power caused by the flow in different elements in the network (e.g. junctions), and the sound power loss across different elements of the network. The general approach considered here is based on two-port theory that divides the duct network into two-port elements. Each element can be described by 2 × 2 scattering matrix where the state variables are the acoustic power flow in both up and downstream directions. Junctions and branching are described by multi-port elements depending on the number of elements connected to this multiport. This algorithm is compared to measurements of HVAC system located in an academic building that shows good agreement. An advantage of this approach is the ability to use the same formalism of the two-port network theory to analyse the acoustic behaviour in both low and high frequency ranges beside the flow distribution and the pressure drop.

1. Introduction

The analysis of sound generation and propagation inside ducts is essential for many applications. The analysis approach depends on the nature of acoustic wave inside a duct that depends on its frequency with respect to the cross dimensions of that duct. Therefore, the frequency range of interest is divided into three different ranges depending on wave behaviour. Each frequency range has its own characteristics and accordingly different modelling techniques.

The low frequency range (plane wave range) is limited by the cut-off frequency of the first mode. In this range, the source is strongly coupled to the system and the acoustic output power varying intensely to system changes. A full modal description of the sound field is needed in this range to simulate the wave propagation accurately. However, only plane waves exist in this range, that make 1D models suitable to describe the behaviour of the acoustic wave [1].

The mid-frequency range starts from the cut-off frequency of plane wave up to three times of it. In this range, plane wave and non-plane waves do exist; and strong coupling between source and system is possible. 1D modelling is not suitable in this range because of existence of non-plane waves. However, other methods can be used (e.g. finite element method and boundary element method) for acoustics analysis in mid-frequency range. Although these methods give acceptable accuracy, they are incapable of covering all mid-frequency range due to its element size limitations (more elements are needed as the frequency gets higher).

It is hard to specify accurately at which frequency the high frequency range starts. A typical assumption is that high frequency range begins from two to three times of cut-off frequency of the first mode. In this range, multiple modes exist and wave length is very small compared to duct size. This makes acoustic waves considered as rays in diffuse field. Hence, there is no coupling between the source and the system, and the acoustic power equals the free field value. The wave characteristics in this range make the power based methodology suitable for modelling, which describe each source by its power and each duct element by its transmission coefficient. The power based models used in this range are often developed based on measurements; for instance, those existing in VDI 2081 [2] and American Society of Heating, Refrigerating and Air-Conditioning Engineers (ASHRAE) [3].

In several applications, the nature of the application determines the frequency range of interest. For instance, in the automotive exhaust systems, the low frequency range analysis is dominating and high frequency analysis is not necessary. However, in other cases, (e.g. HVAC system) the high frequency range analysis is needed. Therefore, it is a necessity to have a combined algorithm to provide analysis in low and high frequency ranges that could be used to predict mid frequency

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range behaviour.

One method provides analysis in both low and high frequency is the Source-Path-Receiver model. In this method; the sound generating device is the source, the path comprises all elements and joints that affect the sound through his way down to the receiver, and the receiver is the site where the sound is heard. This method is widely used in HVAC applications that use the VDI 2081 [2] and ASHRAE [3] standards' models. Typically this algorithm is used to analyse the network from the high frequencies and all the way down to low frequencies [2]. This gives unrealistic results since the power of the source is variant property depending on the acoustic loading from the system. In the high frequency range, the effect of the acoustic loading is minor, but in low frequency range the power output will dramatically change. Therefore, the low frequency analysis should be handled using acoustic pressure models [4].

Another method to describe the sound transmission along the system is called two-port transfer matrix method. This method splits the system into several smaller duct parts (acoustic elements) in which the sound propagation is well defined. In acoustics analysis and in low frequency range, plane waves are assumed to propagate between different elements, and the sound field can be characterized by two state variables. In low frequency range, it is convenient to use pressure and volume velocity. The sound propagation inside each element is analysed separately and higher order modes can exist inside the element [5]. The two-port theory –transfer matrix formalism- was used for many years. It was suited for problems with one preferred direction for the acoustic energy corresponding to a number of cascade coupled two-ports; but it was not suited for more general network with arbitrary connections. An enhanced formalism has been suggested by Eversman [6]. It uses mobility-matrix based on scattering matrix which makes the assembly of complex matrix easy. However, this formalism considers sources as one-port elements at terminations or connected to a joint only. Glav and Åbom [1] enhanced the formalism to include element generated noise as active two-ports. later, Elnady and Elsaadany [7] used the same two-port theory with different state variables, to simulate the flow distribution inside duct networks and couple it to acoustics to get more accurate results in presence of flow. Recently, additional work done by Elnady and Elsahar [8] using the two-port theory with different state variables to model the temperature distribution in duct networks. This will give the ability to couple the temperature distribution with flow and acoustic calculation in order to enhance the accuracy of acoustics results.

In this paper, the two-port theory is used to analyse duct networks in the high frequency range using the formulation used in the low frequency sound pressure analysis and flow analysis. However, this formulation is modified to handle sound power based models and to meet regular HVAC assumptions and boundary conditions such as: no reflection boundary condition, source calculations, and network matrix assembly (see [9] and [10]). This will extend the usage of the two-port theory to cover the high frequency range applications such as HVAC systems.

2. Theoretical background

By assuming linear wave propagation, a building block method will be suitable for analysis. This method splits the network into small elements, and in each element the wave propagation is well defined. Each element is described by 2 × 2 matrix that is so called two-port matrix [11]. This method allows network to be connected in arbitrary fashion, adding source inside the two-port element and connect external sources via one-port elements.

2.1. Description of network elements

In the low frequency range, plane waves only exist. The two-port transfer matrix method is used by characterizing each element with two state variables. Because of strong coupling between source power and network output in low frequency range, the acoustic pressure and volume velocity are used. An alternative representation is the two-port scattering matrix which describes the sound transmission and reflection within one element that can handle sound generation inside the element as well [1].

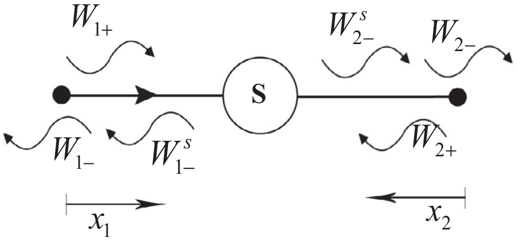

Fig. 1. The two-port element power based variables.

In high frequency range, the coupling between the source and network output is weak. Hence, the acoustic power can be used to describe the two-port element as state variables (see Fig. 1 [4]). The scattering matrix based on acoustic power can be formulated as follows:

$$\begin{bmatrix} W_{1-} \\ W_{2-} \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}_w \cdot \begin{bmatrix} W_{1+} \\ W_{2+} \end{bmatrix} + \begin{bmatrix} W^s_{1-} \\ W^s_{2-} \end{bmatrix} \tag{1}$$

In both VDI 2081 part 1 [2] and ASHRAE [3] standards, the sound attenuation across each element is described by *the Insertion Loss* while the flow generated noise inside each element is described by the generated sound power level from this element. Starting from the given *Insertion Loss* one can get the scattering matrix elements as follows.

$$IL = 10\log_{10}\left(\frac{W_{1+}}{W_{ref}}\right) - 10\log_{10}\left(\frac{W_{2-}}{W_{ref}}\right) \tag{2}$$

where $W_{ref} = 1 \times 10^{-12}$ W, then.

$$W_{2-} = W_{1+}\cdot 10^{-IL/10} \tag{3}$$

The flow generated noise within an element is described in standards by sound power level, and by assuming zero reflection, S_{11} and S_{22} – which represent the reflections- are equal to zero. This assumption along with using Eq. (3) in Eq. (1) gives Eq. (4)

$$\begin{bmatrix} W_{1-} \\ W_{2-} \end{bmatrix} = \begin{bmatrix} 0 & 10^{-IL/10} \\ 10^{-IL/10} & 0 \end{bmatrix} \cdot \begin{bmatrix} W_{1+} \\ W_{2+} \end{bmatrix} + \begin{bmatrix} 0 \\ W^s \end{bmatrix} \tag{4}$$

The same description of the one-port in the low frequency range [1] is used in the high frequency range. Here one-port is described by the acoustic reflection from the source R_s, and Sound Power Source Strength W^s_+, see Fig. 2. By applying zero reflection boundary condition, the power injected to the network through one-port can be described as follows:

$$W = 2W^s_+ - Q \tag{5}$$

where Q is the net power flow through the node [9].

The node is modelled as a multi-port which has number of two-ports connected to it equals to m_n. An illustration of a node is shown in Fig. 3. Note that sign convention for two-port, one-port, and node must be consistent when driving network equations.

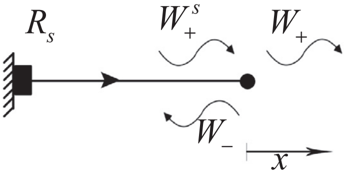

Fig. 2. One-port source in the high frequency range.