Room reflections analysis with the use of spherical beamforming and wavelets

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\textbf{A B S T R A C T}

Beamforming algorithms are widely used nowadays to localize sound sources in several types of environments. In general, the goal is to identify the direction from where a given signal is arriving. It consists of a spatial analysis performed over 2D maps, according to the frequency bands of interest or to a given spectral characteristic. In this paper, a spherical microphone array structure is developed. The signals obtained are input to Wavelet transform, in order to decompose impulse responses for real rooms. The objective is to extend the beamforming array capacity to identify and extract energy peaks from measured room impulse responses. The detected peaks represent sound reflections over the room surfaces, which can be used to describe acoustically the environment studied. With the proposed structure, it is possible to create acoustic maps, discretized in time and frequency, extending the applicability of traditional beamforming to provide directional impulse responses in reverberant spaces. The beamforming combines spherical harmonic decomposition with the Delay-and-Sum algorithm, whose result is post-processed through an Wavelet Transform. The system is applied to the array impulse response and allows to identify the incidence time and direction of the early reflections. The results are compared with those provided by ray-tracing simulation. A very good agreement is obtained for the detection of the first order reflections, besides the physical and spatial constraints of the spherical array developed.

\section{1. Introduction}

In several applications, it is necessary to identify not only the spectral characteristics of sound sources, but also their location or at least the direction of incoming sound waves. In room acoustics, for instance, the knowledge of the sound reflection paths allows the designer to improve the acoustical elements positioning.

There are reports of sound source identification by using array techniques since World War I [1]. One of the most popular techniques is the beamforming with conventional Delay-and-Sum algorithm (DAS), which has been rapidly developed, driven by signal processing hardware and software evolutions.

Beamforming algorithm has been applied mostly for linear or plane microphone arrays, while spherical arrays in acoustics have been employed only after [2,3]. Spherical Harmonic Beamforming (SHB) appears as an alternative for the Delay-and-Sum algorithm, when spherical microphone arrays are used. The SHB is based on spherical harmonic decomposition of the sound pressure over a spherical surface [4], taking into account the spatial sampling, as shown in [5–7].

In general, SHB and DAS algorithms are performed in frequency domain, to avoid the spatial resolution restriction due to time discretization. Considering that frequency domain analysis requires a time window, the sound source shall not vary its spectral characteristics nor its position in space during this time period. Thus, most applications focus on stationary signals [8].

Time-frequency analysis can be performed by Short-Time Fourier Transforms, which decompose the signal into uniform time-frequency slices. Wavelet Transforms (WT) constitute an alternative, by decomposing a signal into non-uniform time × frequency slices [9,10].

In room acoustics, identification of sound reflections and their time of arrival are important tasks. The beamforming can be used to identify such reflections with several methods [11]. However, none of them provides accurate estimates of individual reflections. The objective of this work is to propose a methodology to overcome these difficulties and identify the early reflections experimentally.

In this paper, the early reflections of a room are investigated by generating acoustic maps for several time instants. The proposed method combines a spherical array with post-beamforming processing.
A combination of the Spherical Harmonic Beamforming and the Delay-and-Sum algorithm is applied, according to their working frequency ranges. Beamforming output signals are then decomposed by a wavelet, for creation of detailed acoustic maps, simultaneously in time and frequency. This combination of array types, beamforming methods and wavelet analysis allows to identify direction and time of arrival for early reflections.

In order to evaluate the results of the proposed method, a spherical microphone array was developed, with 20 uniformly spaced microphones. The extracted reflections are compared to those provided by geometric acoustic simulation, using ray-tracing algorithm. The time instants and direction of arrival for reflections up to 3rd order are investigated.

2. Spherical array beamforming

Beamforming is a spatial filter technique that uses microphone arrays and signal processing to control the directivity and the sensitivity of the array with respect to a given direction [12]. This is possible due to the different time signals provided by the spherically spaced sensors with respect to the sound source.

In the Delay-and-Sum method, the array response for a given direction is obtained by applying weights and delays to every sensor signal. The summation of these signals emphasizes the sound coming from that direction. If there is a single arriving sound wave from the focused direction (unique sound source), all delayed signals should be in similar phase and produce a signal with higher amplitude than those coming from other directions. By applying this algorithm for any direction of interest, it is possible to build time-dependent acoustic maps.

Delay-and-Sum can be performed using several algorithms for a variety of applications [1,13]. Specifically for spherical microphone arrays, Spherical Harmonics algorithms can also be employed [3,4]. For this array type, the cut-off frequency for Delay-and-Sum is constrained by the minimal distance between microphones, due to the spatial aliasing effect. Spherical Harmonics algorithm is more adequate to low frequencies and depends strongly on the spherical harmonic order decomposition, i.e., number of sensors and sphere radius. For higher frequencies the Delay-and-Sum algorithm is more appropriate. Therefore, in this work, both beamforming algorithms are performed in parallel, for different frequency bands, according to their most appropriate frequency ranges.

2.1. Delay-and-sum

Considering a plane wave travelling in the direction \( \vec{s} \), the sound pressure at the \( j \)th microphone, \( p_j(t) \), is a function of its position \( \vec{x}_j \) and time \( t \), i.e.:

\[
p_j(t) = p(\vec{x}_j, t) = f(t + \Delta_j(\vec{s})),
\]

where \( c_0 \) is the sound speed and \( \Delta_j \) is the delay of the \( j \)th sensor related to an array reference point. For plane waves, the delay is given by:

\[
\Delta_j(\vec{s}) = \frac{\vec{x}_j \cdot \vec{s}}{c_0}.
\]

The Delay-and-Sum beamforming signal, \( b(\vec{s}_0, t) \), is given by the sum of the \( M \) delayed signals, in respect to a given direction \( \vec{s}_0 \) [1]:

\[
b(\vec{s}_0, t) = \frac{1}{M} \sum_{j=1}^{M} p_j(t - \Delta_j(\vec{s}_0)),
\]

where \( M \) is the number of sensors used.

Eq. (3) can be rewritten in the frequency domain by:

\[
B(\vec{s}_0, \omega) = \frac{1}{M} \sum_{j=1}^{M} P_j(\omega)e^{-i\omega \Delta_j(\vec{s}_0)},
\]

where \( B(\omega) \) and \( P_j(\omega) \) are the Fourier Transforms of \( b(t) \) and \( p_j(t) \), respectively.

By adjusting the delays for a given direction \( \vec{s} \), the resulting signal would be emphasized or attenuated, according to the real sound source location.

A schematic for Delay-and-Sum algorithm is depicted in Fig. 1.

2.2. Spherical harmonic beamforming

Spherical harmonics are functions used to represent the wave equation solution in spherical coordinates. The degree \( n \) and order \( m \) of a spherical harmonic is given by:

\[
Y_n^m(\theta, \phi) = \frac{\sqrt{(2n+1)(n-m)!}}{4\pi(2n)!} L_n^m(\cos \theta)e^{im\phi},
\]

where \( \theta \) and \( \phi \) are the elevation and azimuth angles, respectively, and \( L_n^m(\cos \theta) \) is the associated Legendre polynomial.

The spherical harmonics are orthonormal and satisfy [4]:

\[
\sum_{n=0}^{\infty} \sum_{m=-n}^{n} Y_n^m(\theta', \phi')Y_n^m(\theta, \phi)^* = \delta(\theta-\theta')\delta(\phi-\phi')
\]

and

\[
\int_{\Omega} Y_n^m(\theta, \phi)Y_n^m(\theta, \phi)^* d\Omega = \delta_{nm} \delta_{mm'}.
\]

where \( \delta(a-a') \) is the Dirac’s delta and \( \delta_{mm'} \) is the Kronecker’s delta. The symbol * denotes the complex conjugated.

Considering a function \( f(\theta, \phi) \) which is entirely defined and integrable on the unit sphere surface \( \Omega \), it is possible to decompose this function in spherical harmonics in the following way:

\[
f(\theta, \phi) \equiv \sum_{n=0}^{\infty} \sum_{m=-n}^{n} f(\theta, \phi)Y_n^m(\theta, \phi).
\]

where \( \Omega \) is the \( n \)th degree and \( m \)th order coefficient, which can be obtained by:

\[
f_n = \int_{\Omega} f(\theta, \phi)Y_n^m(\theta, \phi)^* d\Omega.
\]

The advantage of the spherical over the linear or planar arrays is the possibility of having the same beampattern for all directions in space. The ideal beampattern is a Dirac delta, where signals coming from the direction of interest are emphasized and from all other attenuated. The generic spherical harmonic beamforming expression for a direction \( \vec{s} \), represented in terms of \( (\theta, \phi) \) angles, is given by:

\[
B(k\vec{s}, \theta, \phi) = \int_{\Omega} w(k, \theta, \phi, \vec{s}, \theta, \phi) P(k, \theta, \phi)d\Omega,
\]

where \( w \) is a weight that focuses the algorithm into the direction \( (\theta, \phi) \), \( k \) is the wave number, \( a \) is the sphere radius and \( P \) is the Fourier