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# Application of wave field synthesis to active control of highly non-stationary noise

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ABSTRACT

Active Noise Control (ANC) methods have been successfully applied to the cancellation of stationary noise. Classical ANC systems use adaptive filtering techniques to produce a waveform that is opposite, or counter-phase, to the signal noise we would like to cancel. However, when the noise is of short duration, adaptive filtering cannot be used since convergence is not achieved. In this paper, a novel active control technique for non-stationary noise is presented. The method uses wave field synthesis (WFS) for the construction of the canceling waveform. The system is tailored for an outdoor environment. The noise acoustic field is acquired by microphones and processed by a WFS engine to pilot a linear array of secondary sources. Experimental results, obtained from both simulations and true tests, demonstrate that the proposed method is able to diminish the overall noise perceived in the area covered by the system.

## 1. Introduction

Active noise control (ANC) techniques have been widely used to reduce the impact of noise in different environments. Given a source of noise, the aim of an ANC system is that of piloting one or more secondary sources to generate a waveform that is opposite, or in counter-phase, to that of the noise. Thus, thanks to the principle of superposition, the primary source of noise is cancelled. An ANC system is particularly effective when the region where it operates is limited, e.g., in headset, ducts, car and airplane cabins, and is more convenient to diminish low frequency noise than passive methods [1,2].

A classical ANC system is based on adaptive filtering. Its basic components are reference microphones, to acquire the primary source of noise; secondary loudspeakers, to generate the counter-phase waveform; and error microphones, to acquire the resulting signal coming from primary and secondary sources in the area where the system is supposed to be effective. The error signal is used to update the coefficients of adaptive filters that drive the secondary sources. Adaptive algorithms, such as least mean square (LMS), filtered-x LMS (FxLMS) [3] and their extensions [4,5], are capable to catch up the variations of the noisy acoustic field and, consequently, to minimize the variance of the residual acoustic signal. Real-time ANC systems can be effectively exploited in scenarios where the noise can be modeled as a stationary or slowly varying process with respect to the reaction time of the ANC systems [6–8].

When the noise is highly non-stationary, for example, when it appears as an impulse having large-amplitude pressure levels with respect to the background and a very short duration, the adaptive filters within an ANC system may not have enough time to reach convergence, making noise reduction ineffective. Examples of such type of noise are represented by man-made disturbances, such as sounds generated by manufacturing plants, vehicle transit, punching machines or construction sites as well as gunshots in shooting ranges.

In this paper, we use the ANC paradigm, that is generating a counter-phase signal, to cancel an impulse-like noise. Since in the literature the term impulsive noise is often used to indicate heavy-tailed distributed disturbances that affect the operations of adaptive ANC systems [9–14], in order not to generate ambiguities, we also term the primary source signal as *highly non-stationary noise* (HNN) and the proposed scheme as *active HNN control* (AHNNC) system.

To put ideas into a real world scenario, consider as an example of such a noise the one produced by a gunshot within a shooting range. This example allows other objectives of this study to be delineated as follows. The HNN we would like to cancel is a short duration wideband signal. Furthermore, the HNN is not assumed as deterministic, even though several features, both in the time and frequency domain, can be derived by the analysis of few realizations of the noise. The scenario in which the AHNNC system should work is the outdoor and the dimensions of the area in which it should reduce the noise are of the order of tens of meters.

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Since, as it is apparent from the above discussion, adaptive filtering techniques cannot be used, the proposed AHNNC system is based on the theory of wave field synthesis (WFS) to generate the counter-phase secondary signal and cancel the primary HNN, where reference microphones are used to acquire the HNN acoustic field and to pilot the secondary loudspeakers. WFS-based applications are usually designed for constructive acoustic field synthesis [15]: by using appropriate driving signals, an array of secondary sources can be set up to reproduce, in a specific area, the sound as it were generated by one or more virtual primary sources. Unfortunately, WFS theory strictly requires an infinite array of elementary infinitesimal sources, so that several approximations are usually introduced in order to obtain a practical implementation. The idea of using the WFS framework to synthesize a virtual anti-source whose acoustic field is used in a destructive manner has been proposed in [16,17]. In this paper, we extend the previous works by formalizing the problem for the case of HNN and by considering several practical issues that arise when the system operates in a free-field environment. Several experimental results, obtained from both simulations and real tests, are presented to validate the proposed AHNNC system and to assess its performance.

The paper is organized as follows: Section 2 summarizes the concepts of WFS theory. The formalization of the proposed WFS-based ANC system and its implementation issues are presented in Sections 3 and 4, respectively. The results obtained by means of computer simulations as well as by using an experimental testbed are discussed in Section 5. Some concluding remarks are presented in Section 6.

## 2. Overview of wave field synthesis theory

The theory of WFS directly arises from the Kirchhoff-Helmoltz integral [15], which states that the pressure field inside a volume $V$ generated by a distribution of primary acoustic sources can be reproduced by a continuous distribution of elementary secondary sources placed over the boundary of $V$, i.e., the surface $S$. Mathematically, it is equivalent to

$$P(\mathbf{r},\omega) = -\frac{1}{4\pi}\oint_S [G(\mathbf{r}_S|\mathbf{r},\omega)\nabla_{\mathbf{r}_S}P(\mathbf{r}_S,\omega) - P(\mathbf{r}_S,\omega)\nabla_{\mathbf{r}_S}G(\mathbf{r}_S|\mathbf{r},\omega)]\cdot\mathbf{n}_{\mathbf{r}_S}dS. \quad (1)$$

In Eq. (1), $P(\mathbf{r},\omega)$ denotes the pressure field at point $\mathbf{r}\in V$ and having frequency $\omega$;$\mathbf{r}_S \in S$ is a point on the surface $S$;$\mathbf{n}_{\mathbf{r}_S}$ represents the normal to the surface in $\mathbf{r}_S$, pointing inwards $V$;$\nabla_{\mathbf{r}_S}$ is the gradient evaluated in $\mathbf{r}_S$ and $G(\mathbf{r}_S|\mathbf{r},\omega)$ is the appropriate Green function for the considered $N$-D space.

The above relation allows a virtual source (or more virtual sources) to be synthesized for a listener inside $V$, by knowing the values of the pressure field $P(\mathbf{r}_S,\omega)$ and the gradient $\nabla_S P(\mathbf{r}_S,\omega)$ that it induces on the surface $S$; such values are used to excite monopole, i.e., $G(\mathbf{r}_S|\mathbf{r},\omega)$, and dipole, i.e., $\nabla_{\mathbf{r}_S}G(\mathbf{r}_S|\mathbf{r},\omega)$, secondary sources.

In order to be useful in practice, Eq. (1) is usually arranged under some convenient geometrical configuration. A typical example is the *Rayleigh I integral*. Indeed, for a 2-D space, it can be shown that the pressure in the entire half-plane not containing the primary sources can be synthesized by an infinitely extended linear array of elementary monopoles laying on a line $L$, that is

$$P(\mathbf{r},\omega) = -\frac{1}{2\pi}\int_L G(\mathbf{r}_L|\mathbf{r},\omega)\nabla_{\mathbf{r}_L}P(\mathbf{r}_L,\omega)\cdot\mathbf{n}_{\mathbf{r}_L}dL. \quad (2)$$

An analogous formulation for a 3-D space is

$$P(\mathbf{r},\omega) = -\frac{1}{4\pi}\oint_A G(\mathbf{r}_A|\mathbf{r},\omega)\nabla_{\mathbf{r}_A}P(\mathbf{r}_A,\omega)\cdot\mathbf{n}_{\mathbf{r}_A}dA, \quad (3)$$

which states that the pressure can be reproduced by an infinitely extended planar array of elementary monopoles laying on a plane $A$.

In a homogeneous 3-D scenario, if the primary source is a monopole located in $\mathbf{r}_0$ having excitation $S(\omega)$ and if we are mainly interested in the acoustic field on a plane $B$ passing through $\mathbf{r}_0$, a simplified solution can be also considered. The $2D^{1/2}$ *Rayleigh I integral*, proposed in [18], states that the wave field synthesis can be approximated by means of a linear array $L$ laying on $B$, that is

$$P(\mathbf{r},\omega) \approx -g_0\int_L S(\omega)\sqrt{\frac{j\omega|\mathbf{r}_L-\mathbf{r}_0|}{2\pi c}}\cos\varphi_{inc}\cdot G(\mathbf{r}_0|\mathbf{r}_L,\omega)G(\mathbf{r}_L|\mathbf{r},\omega)dL, \quad (4)$$

where $c$ is the speed of sound in the medium; $\varphi_{inc}$ is the angle between the normal to the array line $L$ laying on $B$ and the line passing through $\mathbf{r}_0$ and $\mathbf{r}$. In order to define $g_0$, consider the plane $C$ normal to $B$ and passing through $L$; then

$$g_0 = \sqrt{\Delta_C\mathbf{r}/(\Delta_C\mathbf{r} + \Delta_C\mathbf{r}_0)}, \quad (5)$$

being $\Delta_C\mathbf{r}$ and $\Delta_C\mathbf{r}_0$ the distance between $\mathbf{r}$ and $C$ and between $\mathbf{r}_0$ and $C$, respectively. Since $g_0$ depends on $\mathbf{r}$, the approximation given by the $2D^{1/2}$ Rayleigh I integral is more accurate for listeners on the plane $B$ and placed at a given distance from $L$. Moreover, the approximation becomes increasingly poorer also when $\mathbf{r}$ moves farther from the plane $B$.

## 3. WFS-based AHNNC

In the scenario considered in this paper for the AHNNC system, the primary noise source is represented by an acoustic monopole placed at $\mathbf{r}_0$ in a homogeneous medium and excited by a signal $s(t)$ characterized by a short duration and, thus, by a wideband Fourier transform $S(\omega)$. The source induces a pressure field at the location $\mathbf{r}$, belonging to the area surrounding the source, given by $p(\mathbf{r},t)$, that can be expressed by using its Fourier transform $P(\mathbf{r},\omega)$ as

$$p(\mathbf{r},t) = \frac{1}{2\pi}\int_{-\infty}^{\infty}P(\mathbf{r},\omega)e^{j\omega t}d\omega, \quad (6)$$

The goal of the proposed AHNNC system is reproducing a canceling pressure field $\hat{p}(\mathbf{r},t) = -p(\mathbf{r},t)$ in an $N$-D half-space not containing the source. According to Eq. (6) and the Rayleigh I integral, in a 2-D scenario the canceling linear array placed on the line $L$ should produce

$$\hat{p}(\mathbf{r},t) = -\frac{1}{2\pi}\int_{-\infty}^{\infty} -\frac{1}{2\pi}\int_L G(\mathbf{r}_L|\mathbf{r},\omega)\cdot\nabla_{\mathbf{r}_L}P(\mathbf{r}_L,\omega)\cdot\mathbf{n}_{\mathbf{r}_L}dL\, e^{j\omega t}\, d\omega. \quad (7)$$

For practical purposes, a discrete-sources version of (7) must be considered. Let $\Delta_L$ be the spacing among the secondary sources on $L$. Thus, (7) can be rewritten as

$$\hat{p}(\mathbf{r},t) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\sum_i G(\mathbf{r}_i|\mathbf{r},\omega)\cdot\frac{\Delta_L}{2\pi}\nabla_{\mathbf{r}_i}P(\mathbf{r}_i,\omega)\cdot\mathbf{n}_{\mathbf{r}_i}e^{j\omega t}\, d\omega, \quad (8)$$

where $\mathbf{r}_i$ represents the position of the monopoles on the sampling grids over $L$. Equality in the previous relation strictly holds if $\Delta_L \leqslant c\pi/\omega_{max}$, being $\omega_{max}$ the maximum frequency contained in $S(\omega)$; otherwise, anti-aliasing strategies have to be considered [18]. Eq. (8) states that the acoustic field $\hat{p}(\mathbf{r},t)$ is synthesized by an infinite number of monopoles excited by $\hat{S}_i(\omega)$, given by

$$\hat{S}_i(\omega) = \frac{\Delta_L}{2\pi}\nabla_{\mathbf{r}_i}P(\mathbf{r}_i,\omega)\cdot\mathbf{n}_{\mathbf{r}_i}. \quad (9)$$

If the canceling linear array is used in the 3-D space, according to the $2D^{1/2}$ Rayleigh I integral, the excitations can be rearranged as

$$\hat{S}_i(\omega) = S(\omega)G(\mathbf{r}_0|\mathbf{r}_i,\omega)\sqrt{j\omega}\,\Delta_L\cdot g_0\sqrt{\frac{|\mathbf{r}_i-\mathbf{r}_0|}{2\pi c}}\cos\varphi_{inc}. \quad (10)$$

Similar formulations can be written for the case of a canceling planar array in the 3-D space; however, they are not considered in this paper since planar arrays are assumed to be of difficult deployment in practical applications. Henceforward, we will refer to (10) as the basic formula to synthesize the counter-phase acoustic field.

Several issues related to a practical use of (10) are discussed in the