Applied Acoustics 111 (2016) 1–7

Contents lists available at ScienceDirect

Applied Acoustics

journal homepage: www.elsevier.com/locate/apacoust

Transmission line based struck string model

Jingeol Lee

Department of Electrical Engineering, Paichai University, 155-40 Baejae-ro, Seo-gu, Daejeon 302-735, Republic of Korea

ARTICLE INFO

Article history:
Received 4 August 2015
Received in revised form 1 April 2016
Accepted 4 April 2016
Available online 8 April 2016

Keywords:
Transmission line
Struck string model
Traveling wave
Displacement
Force

ABSTRACT

The analogy of a vibrating string to an electrical transmission line with the correspondences of the displacement to the voltage and the rigid end to the short circuit and the calculations for the hammer–string force in the well known finite difference form of the wave equation for a struck string are applied to build a transmission line based struck string model. The proposed model is validated by showing that the displacements and the forces exerted on the string at the contact with a hammer from the proposed model are consistent with those from the finite difference form.

1. Introduction

The behavior of a vibrating string has been studied in various ways. In the continuous time domain, a string has been modeled with an electrical transmission line based on the fact that oppositely traveling waves exist on both the string and the transmission line [1–3]. In the discrete time domain, a string has been modeled with digital waveguides which consist of two digital delay lines carrying sampled traveling waves [4]. Aside from these models, the wave equation governing the behavior of a flexible string has been solved for various boundary conditions including rigid ends and ways of exciting the string like plucking or striking [5]. As a numerical approach, the wave equations for a plucked, a struck, and a bowed string have been approximated to finite difference forms [6]. Various numerical models of the hammer–string interaction were discussed and the finite difference form of the wave equation for a struck string with stiffness and damping was proposed in a paper by Chaigne and Askenfelt [7].

In transmission line based string models, the displacement and the rigid end of a string had been analogized to the current and the open circuit on a transmission line, respectively [1–3]. However, it was found that the displacement and the rigid end correspond to the voltage and the short circuit through the theory of a transmission line and the circuit simulations, respectively. Based on these discoveries, the transmission line based plucked string model and the circuit based classical guitar model have been built [8,9]. The circuit based classical guitar model integrates the circuit model for a string, that is, the transmission line based plucked string model, with that for a body of guitar. In Kock's transmission line based struck string model, the hammer collision has been modeled with a switch in series with an inductor across a transmission line at the location corresponding to the contact with a hammer. The nonelastic compression of the felt on the hammer was suggested to be implemented by a leak in an additional capacitor in series with the inductor, which is based on the correspondences of the displacement and the rigid end on the string to the current and the open circuit on the transmission line, respectively [1]. Contrary to the above mentioned model, the same correspondences, which were used in the transmission line based plucked string model, and the calculations for the hammer–string force in the well known finite difference form of the wave equation for a struck string are applied to develop a transmission line based struck string model. Since a transmission line is a linear device, the stiffness causing the nonlinearity of a string together with damping as in a paper by Chaigne and Askenfelt are not considered, and thus the finite difference form for a flexible string with no damping by Giordano and Nakanishi is taken as a target the proposed model has to achieve [6,7]. This model is built such that its behavior in terms of the displacement and the force exerted on the string at the contact point are consistent with those from the finite difference form. The proposed model is built and simulated using PSpice.

The paper is organized as follows; the finite difference form of the wave equation for a struck string is reviewed in Section 2. In Section 3, the transmission line based struck string model is proposed, and the model outputs are compared with those from

E-mail address: jingeol@pcu.ac.kr

http://dx.doi.org/10.1016/j.apacoust.2016.04.002
0003-682X/

the finite difference form, and then conclusions are drawn in Section 4.

2. Review of the finite difference form of the wave equation for a struck string

Piano hammers are known to act as nonlinear springs [6]. The hammer–string force can be approximated to be

$$F_h(z_f) = K|z_f|^p \tag{1}$$

where z_f is the amount that the felt covering a hammer is compressed by as the hammer hits a string, K is an effective stiffness constant, and the exponent p is the strength of the nonlinearity. It is known that $p \approx 3$ generally gives a reasonable description of real hammers.

The wave equation for an ideal flexible string with no damping is given by

$$\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2} \tag{2}$$

where $y(x, t)$ is the displacement of the string, and x and c are the distance and the velocity of a wave along the string, respectively. The velocity is given by $c = \sqrt{T/\mu}$ where T is the tension, and μ is the mass per unit length of the string. By sampling time and space of the wave Eq. (2) with a time step, Δt and a spatial step, Δx as $y(x, t) \to y(i\Delta x, n\Delta t) \to y(i, n)$ and then multiplying both sides by $(\mu\Delta x)$ to make each of them the force exerted on an element of the string and finally including the hammer–string force given by Eq. (1), the finite difference form of the wave equation for a struck string is derived as

$$(\mu\Delta x)\frac{y(i,n+1) + y(i,n-1) - 2y(i,n)}{(\Delta t)^2} \approx (\mu\Delta x)c^2\left[\frac{y(i+1,n) + y(i-1,n) - 2y(i,n)}{(\Delta x)^2}\right] + F_h \tag{3}$$

The displacement of the string at time step $n + 1$ is derived from Eq. (3) as

$$y(i,n+1) = 2(1-r^2)y(i,n) - y(i,n-1) + r^2[y(i+1,n) + y(i-1,n)] + \frac{(\Delta t)^2}{\mu\Delta x}F_h \tag{4}$$

where $r = c\Delta t/\Delta x$. The displacements at both ends are set to zero at the beginning of a program of the finite difference form, and are not updated, which corresponds to rigid ends. Newton's second law is applied to describe the motion of the hammer as

$$m_h a_h(n) = -F_h \tag{5}$$

where $a_h(n)$ is the hammer acceleration, and m_h is its mass. Using the Euler method, the position of the hammer, z_h and its velocity, v_h are given as

$$\begin{aligned} z_h(n+1) &= z_h(n) + v_h(n)\Delta t \\ v_h(n+1) &= v_h(n) + a_h(n)\Delta t \end{aligned} \tag{6}$$

For each time step, n, the hammer–string force using Eq. (1) is calculated with $z_f = y_{contact}(n) - z_h(n)$ where $y_{contact}(n)$ is the string displacement at the contact with the hammer. Then, the string displacement at the contact point and those at other locations except both ends are calculated according to Eq. (4) with the updated F_h at the time step, $n - 1$ and with $F_h = 0$, respectively. At the same time, the hammer acceleration, the hammer velocity and position are updated using Eqs. (5) and (6), respectively. Whether the hammer loses contact with the string is decided by monitoring the sign of z_f. When it becomes positive, it is taken as losing contact with sting, and thus the hammer–string force, F_h in Eq. (4) becomes 0. From this moment on, the string displacement for every element is updated using Eq. (4) with $F_h = 0$. On the other hand, the force exerted by the struck string on a bridge is given by

$$F_{bridge} = T\frac{[y(N,n) - y(N+1,n)]}{\Delta x} \tag{7}$$

where N is the number of string elements, and $y(N + 1, n) = 0$ corresponding to the rigid end.

The time evolutions of the hammer–string force and the displacement of the string at the contact point for the initial hammer velocities of $v_h = 0.5$ m/s and $v_h = 4$ m/s are shown in Figs. 1–4, respectively. Figs. 5 and 6 show the magnitudes of FFTs for the forces exerted on the bridge for each case, respectively, from which it is expected that the high frequency harmonics would be richer in the sound with $v_h = 4$ m/s in addition to the increased loudness.

3. Transmission line based struck string model

The correspondences of the displacement to the voltage and the rigid end to the short circuit, which were validated in the transmission line based plucked string model, and the calculations for the hammer–string force in the finite difference form of the wave equation for a struck string, which are reviewed in Section 2, are applied to build a transmission line based struck string model. The calculations are implemented into a circuit in the proposed model. The model for the initial hammer velocity of 0.5 m/s with other parameters to be the same as in the finite difference form is presented in Fig. 7.

In Fig. 7, the transmission lines, T1 and T2 constitute a string, and other parts function as the hammer striking. The time delays of the transmission line, T1 and T2 are set to 0.24 ms and 1.67 ms, respectively, which corresponds to striking the string at a distance one eighth from the left end, and the note of C4 as in the finite difference form. The characteristic impedance of T1 and T2 is set to 1 kΩ arbitrarily. The two NOR gates, U3A and U4B constitute a SR latch whose output depends on whether the hammer keeps contact with the sting. The output of U4B is set to 5 V at $t = 0$ resulting in closing the voltage-controlled switch, S1, by which the hammer–string force starts to be delivered to the string.

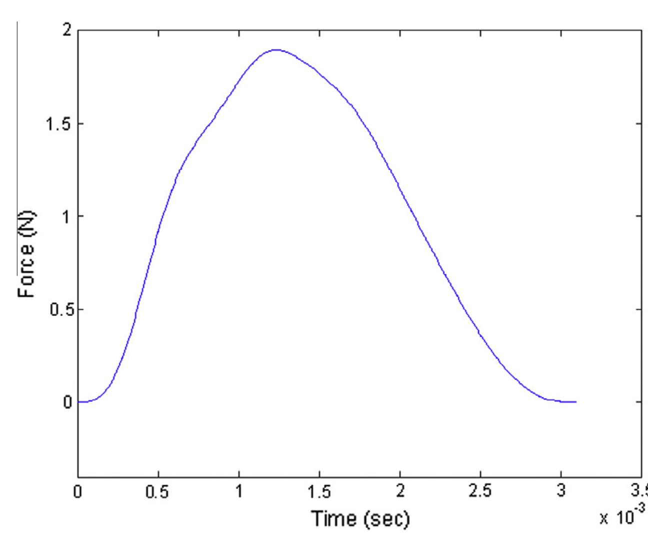

Fig. 1. Hammer–string force for the initial hammer velocity of $v_h = 0.5$ m/s. The length of the string with both ends rigidly fixed is 0.62 m, $\Delta x = 0.62$ mm, $r = 1$, $T = 650$ N, $K = 1.0 \times 10^{11}$ N/m$^{1/3}$, $p = 3$, and $m_h = 3.3$ g. The fundamental vibrating frequency is tuned to 262 Hz corresponding to the C4 note, and the string is struck at a distance one eighth from an end.