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Verification of mathematical formulae applied to overhead stage canopy design

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ABSTRACT

This paper presents a comparison of the experimental research concerning overhead stage canopies with a numerical approach based on selected mathematical models. The numerical predictions are made using the simplified asymptotic curves suggested by Rindel and modified by Skålevik. For singular cases a prediction with detailed calculations based on the Fresnel–Kirchhoff approximation is also given. The aim of the work is to verify proposed algorithms for designing reflective panels as well as to determine the conditions of conducting such procedures. It is shown that based on Rindel's approximation one may determine some substantial information about sound reflection from the panels i.e. the value of upper limit frequency as well as the relative sound reflection level. On the other hand, the lower cut-off frequency should be calculated using Skålevik's model as the value obtained from Rindel's formula is undervalued. Such an approach could be applied to design reflective structures. However, it has some limitations for example for arrays of perturbed symmetry or sparse arrays as well as in the case of non-perpendicular angles of sound wave incidence. Then it may be necessary to apply more accurate numerical models.

1. Introduction

In interiors designed for music and speech performances a fundamental issue is to provide proper sound distribution as well as satisfactory audibility among musicians on the stage. In order to fulfil these requirements in spaces such as concert halls and auditoria, reflective structures suspended over the stage can be introduced (Fig. 1). The basic role of such acoustic panels is to properly transfer the first reflection of a sound wave. Moreover, their presence contribute greatly to favourable spatial sound directing.

So far no general guidelines concerning the design of such reflective structures have been specified. Although architects and designers can base their designs on some scientific publications presenting sample panels [1,2] and take advantage of some mathematical descriptions of sound reflection and diffraction [3,4], a review of the available methods and a verification of some algorithms for designers seem to be invaluable. In the following paper the specification of some procedures and relevant conditions for designing such reflective structures on the basis of mathematical models is ventured.

2. Theoretical background

Presently available studies on the theory of sound reflection allow one to describe this phenomenon in a number of ways. One of the approaches to this problem is formulated by the inhomogeneous Helmholtz equation:

$$\Delta p(\mathbf{r}) + k^2 p(\mathbf{r}) = -q(\mathbf{r}), \tag{1}$$

where $p(\mathbf{r})$ denotes the acoustic pressure dependent only on the spatial variable, $q(\mathbf{r})$ is the function characterising the sound source also dependent on the spatial variable, k is the wave number and Δ is the Laplace operator.

A number of numerical methods can be used to solve the problem given by Eq. (1). One may apply the finite element method (FEM), the finite difference time domain method (FDTD) and the boundary element method (BEM), to name a few [3,5]. However, due to the high computational cost of solving the fundamental equation directly, some simplifications to the model have been introduced.

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Fig. 1. Reflective panels suspended over the stage in ICE Congress Center, Cracow (on the left) and in AGH conference hall, Cracow (on the right).

The high frequency asymptotic method might be used as such an approximation. This approach is based on the decomposition of the total sound field into a direct, reflected and diffracted field. Calculating the first two terms is usually relatively easy as opposed to the diffraction part. This might be described by several different diffraction models, for instance involving multiple-order edge diffraction components [6,7].

The other class of approximations is based on the Kirchhoff approximation (KA) where the integral form of the Eq. (1) is considered and also the separation into the geometrical acoustic and diffraction field is used. The conversion of the surface integral into the line integral is introduced based on Maggi-Rubinowicz transformation [8], therefore the diffraction term is described along the edge of a scatter. This simplification is correct with the assumption that the sound pressure on the surface reflector is doubled with respect to the incident free field. KA has been successfully applied to study the sound reflection from finite rigid plane panels [9]. KA can be simplified further as it is done in the Fresnel–Kirchhoff (F–K) approximation and its modification proposed by Rindel, if an additional assumption that the sound source and the receiver are significantly distant from each other is introduced. In the F–K approximation the integration is done using Fresnel integrals which are numerically more efficient, whereas Rindel simplifies the calculation even further. In this paper, these two approximations have been applied as computationally efficient and sufficiently accurate methods which might be useful during the design process.

One of the fundamental assumptions of the Fresnel–Kirchhoff approximation and its simplification given by Rindel is, as it has been mentioned, that the distances from the reflective panel to the sound source r_0 and to the receiver r are large in comparison with the wavelength and the size of the panel. Moreover, in the F–K approximation, the mutual interactions across the surface are not considered as it is done in the most general case.

The intensity of the reflected sound is expressed here by the diffraction coefficient K. As an example, the formula for this coefficient in x-direction of an orthogonal array of reflective panels (Fig. 2) is presented below [10]:

$$K_1 = \frac{1}{2}\left\{\sum_i^I [C(v_{1,i}) - C(v_{2,i})]^2 + \sum_i^I [S(v_{1,i}) - S(v_{2,i})]^2\right\}, \quad (2)$$

where

$$v_{1,i} = \frac{2}{\sqrt{\lambda \hat{r}}}[e_1 - (i-1)m_1)]\cos\theta, \quad (3)$$

$$v_{2,i} = \frac{2}{\sqrt{\lambda \hat{r}}}[e_1 - 2b_1 - (i-1)m_1)]\cos\theta \quad (4)$$

and I is the total number of rows in the x-direction of the reflective array, $C(v)$ and $S(v)$ are Fresnel integrals, λ is the wavelength, $\hat{r}$ is a characteristics distance given by the equation:

$$\hat{r} = \frac{2rr_0}{r_0 + r} \quad (5)$$

and other symbols are defined in Fig. 2.

Since the application of the Fresnel–Kirchhoff approximation is intricate because of the incorporation of analytically unsolvable Fresnel integrals in the formulae, and also due to the large number of considered points, a simplification proposed by Rindel might be considered. This modification is based on the observation that the most interesting and extreme effects are connected with reflection in the centre and at the edges of the panel. Moreover, according to Eqs. (3) and (4), at high frequencies the v values increase and then the main impact on the reflection is generated by a single panel. On the other hand, at low frequencies the v values decrease and the level of diffraction depends on the area density of an array (not on the geometry of the individual panel). Based on these observations one may approximate the value of the sound level attenuation due to diffraction in the frequency range $f_G \leqslant f \leqslant \mu f_g$ by the formula [12]:

$$\Delta L = 10\log K \cong 20\log\mu, \quad (6)$$

where μ is a relative density of the reflector array, defined as the area of all panels divided by the total area covered by the array. Furthermore, f_G and f_g denote the limit frequencies (lower and upper, respectively) which describe the range of effective sound reflection:

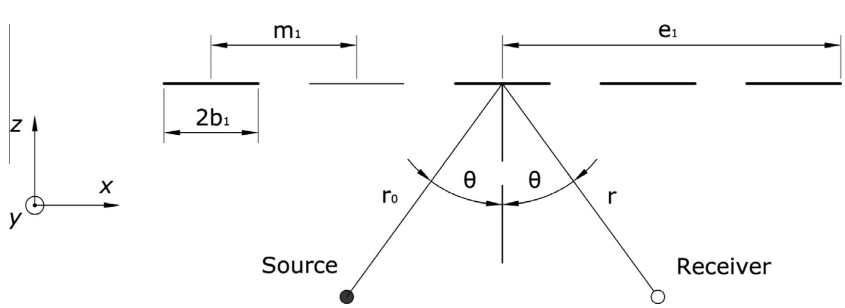

Fig. 2. Section through a reflector array with five rows of reflectors [11].