Applied Acoustics 104 (2016) 76–84

Contents lists available at ScienceDirect

Applied Acoustics

journal homepage: www.elsevier.com/locate/apacoust

An explicit time-domain finite element method for room acoustics simulations: Comparison of the performance with implicit methods

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ARTICLE INFO

Article history:
Received 31 July 2015
Received in revised form 23 October 2015
Accepted 28 October 2015

Keywords:
Room acoustics simulations
Time domain finite element method
Explicit method
Dispersion error
Implicit method

ABSTRACT

This paper presents the applicability of an explicit time-domain finite element method (TD-FEM) using a dispersion reduction technique called modified integration rules (MIR) on room acoustics simulations with a frequency-independent finite impedance boundary. First, a dispersion error analysis and a stability analysis are performed to derive the dispersion relation and the stability condition of the present explicit TD-FEM for three-dimensional room acoustics simulations with an infinite impedance boundary. Secondly, the accuracy and efficiency of the explicit TD-FEM are presented by comparing with implicit TD-FEM using MIR through room acoustics simulations in a rectangular room with infinite impedance boundaries. Thirdly, the stability condition of the explicit TD-FEM is investigated numerically in the case with finite impedance boundaries. Finally, the performance of the explicit TD-FEM in room acoustics simulations with finite impedance boundaries is demonstrated in a comparison with the implicit TD-FEM. Although the stability of the present explicit TD-FEM is dependent on the impedance values given at boundaries, the explicit TD-FEM is computationally more efficient than the implicit method from the perspective of computational time for acoustics simulations of a room with larger impedance values at boundaries.

1. Introduction

Recently, the applicability and the practicality of the wave based acoustics simulation techniques such as finite element method (FEM), boundary element method and finite difference time domain method are gradually increasing in room acoustics design, with the rapid progress of computer technology [1]. Among them, FEM is a powerful tool for predicting a sound field in a room with complex boundary conditions. Because FEM is frequently said to be computationally expensive for room acoustics simulations, the application range is restricted to low-frequency regions in general, but the situation is changing quickly with the development of efficient methods. Dispersion reduction methods such as high order finite elements (FEs) [2,3], Krylov subspace iterative methods [4] and parallel computation methods are examples of techniques to increase the efficiency of the FEM. Thanks to the FEM using these techniques, room acoustics simulations at high frequencies have recently become within the scope of the analysis if the volume of architectural space is relatively small [5].

The FEM can analyze a sound field in a room in both frequency and time domains [6–9]. Because a time-domain FEM (TD-FEM) calculates an impulse response of a room directly in a time domain, it is attractive from the perspective of room acoustics evaluations such as the visualization, the auralization of sound fields and the calculation of room acoustical parameters. The authors have also developed some efficient TD-FEM for large-scale room acoustics simulations with many degrees of freedom (DOF), and the applicability has been presented by several room acoustics simulations such as concert halls and reverberation chambers [5,10–12]. To analyze a sound field efficiently, the TD-FEM uses a high order FEs called hexahedral 27-node spline acoustic elements [13,5] or low-order FEs called hexahedral 8-node elements with modified integration rules (MIR) [15] as well as the use of iterative methods [4] and parallel computation methods [14]. Here, MIR is a simple dispersion reduction technique to reduce an inherent dispersion error coming from the spatial and time discretizations of a computational domain. Although the TD-FEM is based on an implicit method, it can be considered as efficient because the large-scale linear system of equations with a sparse matrix at each time step can solve easily using an iterative method with the good convergence property [5,10,11].

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http://dx.doi.org/10.1016/j.apacoust.2015.10.027
0003-682X/

On the other hand, there exists an explicit TD-FEM with MIR [15], in which fourth-order accuracy with respect to the dispersion error can be obtained for the idealized case using square or cubic FEs. The explicit TD-FEM is also very attractive for realizing an efficient room acoustics simulation because it does not need a solution of a linear system of equations at each time step. With this advantage, it might be computationally more efficient than the implicit TD-FEM and might become an alternative method for room acoustics simulations. However, it has not been applied to room acoustics simulations so far, and the formulation including a dissipation term for treating an absorption at boundaries has not been presented in the literature [15], which is important for room acoustics simulations. More recently, in Ref. [16], the authors presented an explicit TD-FEM using MIR with the dissipation term for room acoustics simulations, and showed the efficiency in term of computational time over an implicit TD-FEM using MIR. However, this study was conducted in limited numerical conditions without a dispersion error analysis and a stability analysis, as a first stage of the research. Therefore, to properly use the method on room acoustics simulations, the applicability of explicit TD-FEM and the performance over implicit TD-FEM need to be examined further.

In this paper, the applicability of the present explicit TD-FEM with the dissipation term on room acoustics simulations is discussed in more detail, including the three-dimensional dispersion error analysis and the stability analysis. The purpose of this paper is to show the accuracy and efficiency of the explicit TD-FEM over the implicit TD-FEM using MIR on room acoustics simulations with a frequency-independent finite impedance boundary. To this end, we conducted theoretical and numerical investigations in a step-by-step manner. First, in Section 3, to show the basic discretization error property of explicit TD-FEM in three-dimensions, a three-dimensional dispersion error analysis is performed, in which we consider an idealized condition for theoretical analysis, which is a plane wave propagation in a free field. This section also includes the comparison of dispersion property between explicit and implicit TD-FEM. Secondly, in Section 4, the theoretical findings are confirmed in room acoustics simulations with infinite impedance boundaries by a comparison between numerical solution and analytical solution because the dispersion error analysis does not consider sound wave propagations in a closed sound field. Thirdly, for more practical problems with frequency-independent finite impedance boundaries, the stability of explicit TD-FEM is examined numerically in Section 5. Finally, the performance of explicit TD-FEM over the implicit TD-FEM on room acoustics simulations with finite impedance boundaries is discussed in Section 6, to show the applicability. Note that numerical investigations are limited to rectangular rooms modeled by the cubic FEs because the explicit TD-FEM can achieve the fourth-order accuracy for the use of only cubic FEs.

2. Theory

2.1. Implicit TD-FEM using MIR and iterative method

We consider a closed sound field with a rigid boundary, a vibration boundary, and an impedance boundary governed by the wave equation. By introducing the FE approximations to sound pressure and weight function in the weak form derived from the wave equation, the semi-discretized matrix equation for the closed sound field is obtainable as

$$\boldsymbol{M}\ddot{\boldsymbol{p}} + c_0^2\boldsymbol{K}\boldsymbol{p} + c_0\boldsymbol{C}\dot{\boldsymbol{p}} = \boldsymbol{f}, \tag{1}$$

where $\boldsymbol{M}$, $\boldsymbol{K}$, and $\boldsymbol{C}$, respectively, denote the global mass matrix, the global stiffness matrix, and the global dissipation matrix. Further, $\boldsymbol{p}$, $\boldsymbol{f}$, c_0, respectively, denote the sound pressure vector, the external force vector, and the speed of sound. The symbols $\cdot$ and $\cdot\cdot$ respectively signify first-order and second-order derivatives with respect to time. The implicit method solves the above second-order ordinary differential equation (ODE) by using a direct time integration method. In this paper, an efficient formulation for large-scale analyses with many degree of freedom is used to solve the Eq. (1) [11]. In the formulation, a Krylov subspace iterative method called Conjugate Gradient (CG) method is used to solve the large-scale linear system of equations at each time step efficiently. Furthermore, 8-node hexahedral FEs and Newmark β method [17] are respectively used for spatial and time discretizations, with MIR. The MIR is a simple method to reduce the dispersion error, in which numerical integration points of Gauss–Legendre rule in calculations of the element stiffness matrix $\boldsymbol{k}_e$ and the element mass matrix $\boldsymbol{m}_e$ are modified from conventional points based on the dispersion error analysis. The modified integration points for TD-FEM with 8-node hexahedral FEs and Newmark β method are given as [15]

$$\alpha_k = \pm\sqrt{\frac{2}{3}}, \quad \alpha_m = \pm\sqrt{\frac{2}{3} + \left(\frac{1}{3} - 4\beta\right)\tau^2}, \tag{2}$$

where α_k and α_m represent the numerical integration points in $\boldsymbol{m}_e$ and $\boldsymbol{k}_e$, respectively. τ and β represent the courant number $c_0\Delta t/h$ and a parameter related to the accuracy and the stability in Newmark β method.

Various Newmark schemes have been used with the different values of parameter β [17,18]. This paper uses two Newmark β methods called the constant average acceleration (CAA) method with $\beta = 1/4$ and the Fox–Goodwin (FG) method with $\beta = 1/12$. For a three-dimensional analysis using cubic FEs, the stability condition of the implicit TD-FEM used here is given as [11]

$$\Delta t_{\text{crit}} \leqslant \frac{h}{c_0} \quad \text{(CAA)}, \tag{3}$$

$$\Delta t_{\text{crit}} \leqslant \frac{h}{\sqrt{3}c_0} \quad \text{(FG)}, \tag{4}$$

where Δt_{crit} and h respectively represent the critical time interval and the element length of cubic FEs. The dispersion error, which is defined as the difference between the exact speed of sound c_0 and approximate speed of sound c^h, of the implicit TD-FEM with CAA for the idealized case using cubic FEs can be estimated by [11]

$$\frac{|c_0 - c^h|}{c_0} = \frac{k^4h^4}{1440}[A_1 + A_2 + A_3 + A_4], \tag{5}$$

with

$$\begin{aligned} A_1 &= 3 - 3\tau^4 - \chi_i\cos^2\theta\sin^2\theta, \\ A_2 &= \chi_i\cos^2\phi\sin^2\phi\cos^2\theta\sin^2\theta, \\ A_3 &= -\chi_i\cos^2\phi\sin^2\phi\sin^2\theta, \\ A_4 &= 9\cos^2\phi\sin^2\phi\cos^2\theta\sin^4\theta, \\ \chi_i &= 9 - 20\tau^4, \end{aligned} \tag{6}$$

where k, θ and ϕ respectively represent the wave number, elevation and azimuth in a spherical coordinate system. For the implicit TD-FEM with FG, the dispersion error is [11]

$$\frac{|c_0 - c^h|}{c_0} = \frac{k^4h^4}{480}[\cos^6\phi\sin^6\theta + \sin^6\phi\sin^6\theta + \cos^6\theta - \tau^4]. \tag{7}$$

Note that only the implicit TD-FEM with FG can achieve the fourth-order accuracy even for the use of rectangular FEs.