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# A comparison of impedance models for the inverse estimation of the non-acoustical parameters of granular absorbers

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ABSTRACT

An impedance model relates acoustic properties of porous materials with non-acoustical parameters. Although such parameters can be measured, specific equipment is required for each of them, so that numerical methods have been proposed for estimating their value from a more manageable measurement of the normal incidence absorption coefficient in an impedance tube. This inverse procedure requires both an impedance model and an inversion technique. This paper compares four impedance models, Miki, Hamet–Berengier, Johnson–Allard–Champoux and Champoux–Stinson, when Simulated Annealing is used for the inverse estimation of the non-acoustical parameters of three granular materials, consisting of packings of small spherical glass beads of different diameters. Some of these parameters have also been measured, so that they can be compared with these estimated by the proposed inverse method.

## 1. Introduction

Sound absorbing materials are profusely used in Acoustics for increasing the transmission loss across multilayer walls, decreasing the reverberation in enclosures, or dissipating the noise produced by diverse sources [1]. Most of sound absorbing materials are porous-type, which consist of a solid (rigid or flexible) skeleton and air in between. Sound absorption in porous materials is produced by the particle velocity and thermal gradients between the solid and gas phases of the material. Based on their microstructural configuration, porous absorbing materials can be classified as cellular (foams), fibrous (mineral wools) or granular (aquarium gravel, rubber crumbs) [2]. The most used sound absorbing materials are synthetic foams and fibrous wools since they offer an excellent acoustical performance at a moderate cost. Nowadays, however, there is a trend toward recycled materials, which are of the granular type, and may come from the waste generated in other production plants [3]. A clear example of this trend is the use as absorbing material of rubber crumbs coming from the recycling of out-of-use tyres [4].

The acoustical performance of an absorbing material can be predicted through an impedance model that relates acoustical variables, such as the complex characteristics impedance, $Z_s$, and the complex wave number, $k_s$, with non-acoustical parameters, such as air flow resistivity, $\sigma$, porosity, $\phi$, or tortuosity, $q^2$. A variety of empirical, phenomenological, and microstructural impedance models have been proposed [5,6]. Empirical models provide regression equations for the acoustical variables based on experimental data. The Delany–Bazley [7] and Miki [8] models, fruitfully used in modeling cellular and fibrous materials, as well as the Voronina–Horoshenkov [9] model, recommended in the modeling of loose granular materials, are of the empirical type. The phenomenological approach models the material as a compressive fluid where viscous, due to particle velocity gradients, and thermal, due to thermal gradients, dissipation occurs. The Hamet–Berengier model [10], successfully used in the propagation of sound through porous pavements, is of the phenomenological type. In recent years, substantial effort has been spent in the development of microstructural models that describe the propagation of sound waves across the porous material. The Johnson–Allard–Champoux [6] and the Champoux–Stinson [11] models are of the microstructural type. In general, the more sophisticated the model is, the more non-acoustical parameters are required.

In an ordinary use of a sound absorbing model (direct modeling), Fig. 1, the non-acoustical parameters are first measured and then entered into the impedance model equations to obtain its sound absorption coefficient as a function of frequency. However, some of the parameters are difficult to measure, and specific equipment is required to measure each of the non-acoustical parameters. Some authors have proposed a multiscale approach to compute these parameters from specific finite-element analyses

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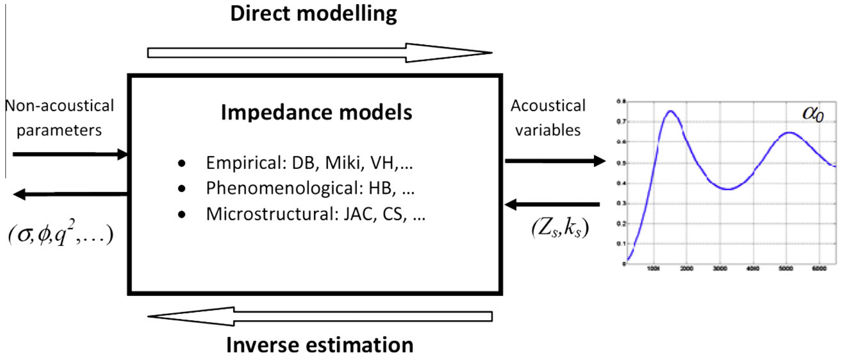

**Fig. 1.** Direct and inverse modeling in absorbing materials. The curve at the right is the normal incidence absorption coefficient.

based on realistic representations of the actual microstructure of the porous material [12]. Alternatively, they can be inversely estimated from the measurement of the sound absorption coefficient at normal incidence [13,14]. As illustrated in Fig. 1, the normal incidence absorption coefficient, as routinely measured in an impedance tube using the transfer function method [15], is used to estimate the non-acoustical parameters. This inverse approach requires the assumption of an impedance model and an inversion technique. The objective of the inversion technique is to obtain the non-acoustical parameters that minimize the difference between the measured and modeled absorption curves. Genetic [13], Simulated Annealing [14], and non-lineal best fitting algorithms [13] have been used to estimate the non-acoustical parameters of porous materials and microperforated panels [15].

The main aim of this paper is to carry out a comparison of impedance models in the inverse estimation of the non-acoustical parameters of a granular material consisting of random packings of identical small glass beads. Simulated Annealing (SA in the following) will be assumed as the inversion technique due to the excellent performance in previous inverse problems [14,16]. The three-parameters Miki model, the phenomenological Hamet–Berengier model (HB in the following), and the microstructural Jo hnson–Allard–Champoux (JAC in the following) and Champoux–Stinson (CS in the following) models will be compared. All these models share the three following parameters: air flow resistivity, $\sigma$, porosity, $\phi$, and tortuosity, $q^2$. JAC and CS are five-parameters models. JAC uses, besides the above mentioned three parameters, the viscous, $\Lambda$, and thermal, $\Lambda'$, characteristics lengths. CS uses instead the viscous, $s_p$, and thermal, $s_k$, shape factors.

Non-acoustical parameters of loosely packed small spheres have been measured by Allard et al. [17]. Umnova et al. [18,19] theoretically predicted these parameters using a cell model with an adjustable cell radius which allows for hydrodynamic interactions between the spheres. Gasser et al. [20] predicted the acoustical properties of regular packing of small hollow beads by means of homogenization and FEM computations. Zielinski [12] applied also FEM techniques to study the influence of the regular sphere packings in the computation of the non-acoustical parameters. Therefore, there exist abundant results in the literature to which the results provided by the inverse estimation proposed in this paper can be compared with.

The paper is organized as follows. The Miki, HB, JAC and CS impedance models are reviewed first in Section 2. The inverse estimation of the non-acoustical parameters by Simulated Annealing is examined then in Section 3. Section 4 describes in more detail the granular absorber, the measurement of some of its non-acoustical parameters and absorption coefficient, and the estimation of such non-acoustical parameters by SA. Finally, the main conclusions of this study are outlined in Section 5.

## 2. Impedance models

Assuming a time dependence $exp(-i\omega t)$, the input acoustic impedance to an absorbing layer of thickness $d$ backed by a rigid wall is [1]

$$Z_i(\omega) = Z_s \, coth[-ik_s d], \tag{1}$$

where $Z_s$ is the complex characteristic acoustic impedance and $k_s$ the complex wave number of the absorbing material. Once the input acoustic impedance, $Z_i$, is known, the normal incidence reflection, $R$, and absorption, $\alpha_0$, coefficients can be calculated from

$$R = \frac{Z_i - Z_0}{Z_i + Z_0}, \tag{2}$$

$$\alpha_0 = 1 - |R|^2, \tag{3}$$

where $Z_0$ is the air characteristic impedance. In the following, the equations for the complex characteristic acoustic impedance and the complex wave number are given for each impedance model. The common non-acoustic parameters for the four models are $\sigma$ the air flow resistivity, $\phi$ the porosity, and $q^2$ the tortuosity (notice that $\alpha_\infty$ is used also by some authors for tortuosity).

### 2.1. The empirical model of Miki

The empirical equations for the Miki model are

$$\begin{aligned} Z_s &= \tfrac{q}{\phi}\left\{1 + 0.07\left(\tfrac{f}{\sigma_e}\right)^{-0.632} + 0.107i\left(\tfrac{f}{\sigma_e}\right)^{-0.632}\right\} \\ k_s &= \tfrac{\omega q}{c_0}\left\{1 + 0.109\left(\tfrac{f}{\sigma_e}\right)^{-0.618} + 0.160i\left(\tfrac{f}{\sigma_e}\right)^{-0.618}\right\}, \end{aligned} \tag{4}$$

where

$$\sigma_e = \frac{\phi}{q^2}\sigma. \tag{5}$$

### 2.2. The phenomenological model of Hamet and Berengier

The HB model is characterized by the equations

$$\begin{aligned} Z_s &= \frac{1}{\phi}\sqrt{\rho_g(\omega)K_g(\omega)} \\ k_s &= \omega\sqrt{\rho_g(\omega)/K_g(\omega)} \end{aligned} \tag{6}$$

where

$$\begin{aligned} \rho_g(\omega) &= \rho_0 q^2\left(1 + \frac{if_\mu}{f}\right) \\ K_g(\omega) &= \gamma P_0\left(1 + \frac{(\gamma - 1)}{1 - \frac{if}{f_\theta}}\right)^{-1} \end{aligned} \tag{7}$$