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Hybrid uncertainty propagation of coupled structural–acoustic system with large fuzzy and interval parameters

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ABSTRACT

Based on the finite element framework and uncertainty analysis theory, this paper proposes a first-order subinterval perturbation finite element method (FSPFEM) and a modified subinterval perturbation finite element method (MSPFEM) to solve the uncertain structural–acoustic problem with large fuzzy and interval parameters. Fuzzy variables are used to represent the subjective uncertainties associated with the expert opinions; whereas, interval variables are adopted to quantify the objective uncertainties with limited information. By using the level-cut strategy and subinterval methodology, the original large fuzzy and interval parameters are decomposed into several subintervals with small uncertainty level. In both FSPFEM and MSPFEM, the subinterval matrix and vector are expanded by the Taylor series. The inversion of subinterval matrix in FSPFEM is approximated by the first-order Neumann series, while the modified Neumann series with higher order terms is adopted in MSPFEM. The eventual fuzzy interval frequency responses are reconstructed by the interval union operation and fuzzy decomposition theorem. A numerical example evidences the remarkable accuracy and effectiveness of the proposed methods to solve engineering structural–acoustic problems with hybrid uncertain parameters.

1. Introduction

Finite element method (FEM) has been widely used for numerical simulation of complex systems given the deterministic modeling parameters [1]. But for most engineering cases, various uncertainties associated with material properties, external loads and geometric dimensions are ubiquitous [2]. From the numerical computational methods, the uncertainty propagation can be grouped into three categories: probabilistic method, possibilistic approach, and anti-optimization method [3]. From the uncertainty modeling methods, the techniques to quantify the system uncertainties can be grouped into probabilistic model, interval analysis and fuzzy theory [4]. When substantial statistical information exists, the probabilistic modeling approaches, such as Monte Carlo simulation, spectral analysis method and stochastic collocation method, can be considered as the most valuable strategies to predict the uncertain system responses [5,6]. Unfortunately, for many complex practical problems, the objective information to define the precise probability density functions of the uncertain parameters may not be easily available in the early stage of numerical analysis. In such situation, the non-probabilistic approaches, such as interval model [7] and fuzzy set [8] can be adopted.

Interval analysis is perfectly appropriate to deal with the uncertain problems where only the lower and upper bounds of input parameters are well-defined [9]. According to the perturbation theory and interval algorithms, Qiu et al. [10] presented the interval parameter perturbation method to solve the structural mechanical problems, where the interval matrix inverse is calculated by the first-order Neumann series. Resorting to its small computational cost and easily guaranteed convergence condition, the interval perturbation method has been widely applied in engineering problems [11,12]. However, due to the unpredictable effect of neglecting high-order terms in Neumann series, the traditional perturbation method will not be effective for the cases with large uncertainty level. By employing the subinterval theory and SMW formula, Xia et al. [13,14] extended the perturbation method to the problem with large interval parameters, and obtained acceptable computational accuracy. Fuzzy set theory is another efficient category to model the system uncertainties based on the subjective opinions. This description can be implemented by the level-cut strategy, which subdivides the membership function range into a number of cut levels and transforms the original fuzzy variables into some interval ones [15]. Up to now, there are two main kinds of approaches for the fuzzy analysis. The first one is known as the global optimization based approach, in which two optimization

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problems aiming at the maximal and minimal values of the output quantity will be solved for each cut level [16,17]. The computational accuracy is high, but the huge computational burden for the large number of optimization problems embarrasses its application in engineering. The second one is known as the interval algebra based approach, where the fuzzy variable is considered as an interval variable for each cut level, and the uncertainty is predicted by using the classical interval arithmetic [18,19].

For a thin-walled structure, the interaction between the vibrating structure and the acoustic field cannot be ignored, and the study on coupled structural–acoustic systems with deterministic parameters has undergone a rapid development [20]. Considering the unavoidable uncertainties caused by the model inaccuracies and system complexities, the nondeterministic models and numerical methods are more feasible [21]. By using the perturbation method and change-of-variable technique, Xia and Yu [22] investigated the probabilistic characteristics of the stochastic structural–acoustic system. Based on the Lanczos method, an efficient computational procedure was proposed for the random structural–acoustic simulations [23]. Besides, numerous researches using the pure interval or fuzzy models have been carried out for the uncertain structural–acoustic problems [24,25]. However, in practical engineering, different kinds of uncertain parameters may exist simultaneously. Many approaches for the hybrid problems with random and interval parameters have been proposed [26,27], but the hybrid framework integrating the interval analysis and fuzzy theory is still in the preliminary stage [28,29].

This paper aims at extending and developing the subinterval perturbation method to solve the hybrid fuzzy and interval structural–acoustic problem. The FEM model of the coupled structural–acoustic system is firstly established in Section 2. Then we review the basic theory of level-cut strategy and subinterval methodology in Section 4. Subsequently, two efficient subinterval perturbation finite element methods are presented in the next two sections. An engineering example is provided to demonstrate the effectiveness and accuracy of the proposed methods, and we conclude the paper with a brief discussion at last.

2. FEM equation for the coupled structural–acoustic system

Considering the interaction between the vibrating structure and the acoustic field, the coupled structural–acoustic system has been widely used in submarine, vehicle body and aircraft. Fig. 1 depicts a structural–acoustic model, which is consisted of the structural domain Ω_s, interior acoustic domain Ω_a and coupled interface Ω_{sa}.

In the finite element analysis without considering the structural damping, the FEM equation of the shell structure can be expressed as

$$\mathbf{M}_s\ddot{\mathbf{u}} + \mathbf{K}_s\mathbf{u} = \mathbf{F}_s + \mathbf{F}_b \tag{1}$$

where $\mathbf{u}$ and $\ddot{\mathbf{u}}$ stand for the nodal displacement vector and acceleration vector of the structural responses; $\mathbf{M}_s$, $\mathbf{K}_s$, $\mathbf{F}_s$ and $\mathbf{F}_b$ are the structural mass matrix, structural stiffness matrix, structural surface load vector and body load vector, respectively.

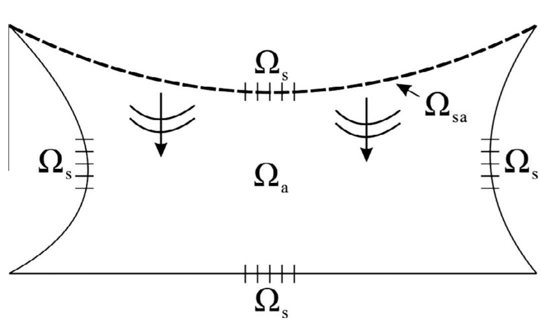

Fig. 1. Coupled structural–acoustic system.

Similarly, the FEM equation of the acoustic field neglecting the acoustic damping can be expressed as

$$\mathbf{M}_a\ddot{\mathbf{p}} + \mathbf{K}_a\mathbf{p} = \mathbf{F}_a + \mathbf{F}_q \tag{2}$$

where $\mathbf{p}$ is the nodal pressure vector of the acoustic responses; $\mathbf{M}_a$, $\mathbf{K}_a$, $\mathbf{F}_a$ and $\mathbf{F}_q$ stand for the acoustic mass matrix, acoustic stiffness matrix, acoustic surface load vector and additional load vector, respectively.

If the structural domain and acoustic domain are considered as a coupled system, the interface should satisfy the continuity conditions. Then the acoustic load applied on the shell structure $\mathbf{F}_s$ in Eq. (1) and the structural load applied on the acoustic fluid $\mathbf{F}_a$ in Eq. (2) can be rewritten as the following forms

$$\begin{aligned} \mathbf{F}_s &= -\int_{\Omega_{sa}} \mathbf{N}_s^T \sigma d\Omega = \int_{\Omega_{sa}} \mathbf{N}_s^T p\mathbf{n} d\Omega = \left(\int_{\Omega_{sa}} \mathbf{N}_s^T \mathbf{N}_a d\Omega\right)\mathbf{p} = \mathbf{C}\mathbf{p} \\ \mathbf{F}_a &= \rho_a \int_{\Omega_{sa}} \mathbf{N}_a^T \ddot{u}_a d\Omega = \rho_a\left(\int_{\Omega_{sa}} \mathbf{N}_a^T \mathbf{N}_s d\Omega\right)\ddot{\mathbf{u}} = \rho_a \mathbf{C}^T\ddot{\mathbf{u}} \end{aligned} \tag{3}$$

where $\mathbf{N}_s$ and $\mathbf{N}_a$ stand for the Lagrange shape function vectors of the structural and acoustic isoparametric elements; σ denotes the surface traction; $\mathbf{n}$ is the normal vector of the interface; ρ_a is density of the acoustic fluid; $\ddot{u}_a$ represents the normal acceleration of the acoustic fluid contacting the shell structure, and $\mathbf{C}$ is introduced as the coupled matrix.

Assuming that the external excitation is time harmonic, the acceleration vectors can be expressed as $\ddot{\mathbf{u}} = -\omega^2\mathbf{u}$ and $\ddot{\mathbf{p}} = -\omega^2\mathbf{p}$, where ω denotes the angle frequency. Thus, based on the combination of Eqs. (1) and (2), the FEM equation for the coupled structural–acoustic system can be denoted as

$$\mathbf{AT} = \mathbf{F} \tag{4}$$

where

$$\mathbf{A} = \begin{pmatrix} \mathbf{K}_s - \omega^2\mathbf{M}_s & -\mathbf{C} \\ \rho_a\omega^2\mathbf{C}^T & \mathbf{K}_a - \omega^2\mathbf{M}_a \end{pmatrix} \quad \mathbf{T} = (\mathbf{u} \quad \mathbf{p})^T \quad \mathbf{F} = (\mathbf{F}_b \quad \mathbf{F}_q)^T \tag{5}$$

For the engineering structural–acoustic problems, due to the vaguely defined system characteristics, insufficient information and judgment subjectivity, uncertainties in material properties, external loads and geometric dimensions are unavoidable. In this paper, the uncertainties whose membership functions can be defined subjectively based on the expert opinions are modeled as m fuzzy parameters

$$\boldsymbol{\alpha}^F = (\alpha_i^F)_m = (\alpha_1^F, \alpha_2^F, \ldots, \alpha_m^F) \tag{6}$$

And the uncertainties whose lower and upper bounds can be determined by the limited information are quantified as n interval parameters

$$\begin{aligned} \boldsymbol{\beta}^I &= (\beta_i^I)_n = \left([\underline{\beta}_i, \bar{\beta}_i]\right)_n = (\beta_i^c + \Delta\beta_i^I)_n = (\beta_i^c + \Delta\beta_i\delta_i^I)_n \\ &= \boldsymbol{\beta}^{\mathbf{c}} + \Delta\boldsymbol{\beta}\delta^I \end{aligned} \tag{7}$$

where $\underline{\beta}_i$ and $\bar{\beta}_i$ are the lower and upper bounds of the interval variable β_i^I; $\beta_i^c = (\bar{\beta}_i + \underline{\beta}_i)/2$ and $\Delta\beta_i = (\bar{\beta}_i - \underline{\beta}_i)/2$ are called the midpoint and the radius, respectively; the transition parameter δ_i^I denotes a standard interval $\delta_i^I = [-1, 1]$.

Obviously, the structural–acoustic stiffness matrix $\mathbf{A}$ and equivalent load vector $\mathbf{F}$ in Eq. (4) become fuzzy interval matrix and vector with respect to the fuzzy vector $\boldsymbol{\alpha}^F$ and interval vector $\boldsymbol{\beta}^I$. Thus, the structural–acoustic FEM equation with hybrid fuzzy and interval parameters can be written as

$$\mathbf{A}(\boldsymbol{\alpha}^F, \boldsymbol{\beta}^I)\mathbf{T}(\boldsymbol{\alpha}^F, \boldsymbol{\beta}^I) = \mathbf{F}(\boldsymbol{\alpha}^F, \boldsymbol{\beta}^I) \tag{8}$$