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Use of a reciprocity technique to measure the radiation efficiency of a vibrating structure

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ABSTRACT

The reciprocity principle is well-known and has many applications in acoustics and vibro-acoustics. This paper discusses a reciprocity measurement method to determine the radiation efficiency of a vibrating structure. The method comprises two steps: (i) measurements of the acceleration response of the structure induced by a sound field in a reverberation chamber and (ii) measurements of the spatially-averaged squared transfer mobility of the structure. The approach is more flexible than a direct method and has the advantage that no shaker is required to excite the structure in the acoustic measurements. To demonstrate the applicability of this method, experiments were conducted on rectangular flat plates, on two components of a railway track test-rig and on three different built-up structures. For the plates and the railway rig components, comparisons are also made with theoretical models. It is shown that the measured results for each arrangement obtained using this reciprocity method provide good agreement with conventional direct measurements and with theoretical modelling. However, in most of the examples presented, the direct method has been found to be less practical and sometimes even less accurate than the reciprocal one, mostly due to the structure-shaker connection and to the inherent uncertainty of acoustic intensity measurements.

1. Introduction

The radiation of sound from a vibrating structure can be defined in terms of its radiation efficiency, which is the sound power normalised by the surface area, the spatially-averaged surface velocity and the characteristic acoustic impedance of the medium. To measure this quantity two separate steps are required: in the first the acoustic power radiated by the vibrating body is measured, while in the second the spatially-averaged mean-square normal velocity is determined. Either the same excitation must be applied in each case, or the responses should be normalised in each case by the corresponding mean-square force amplitude.

A number of conventional methods are available that can be used to measure the radiated sound power. These include methods based on sound pressure measurements, viz. the free field method [1] or the reverberation room method [2], and the sound intensity method [3]. However, each of these 'direct' methods has limitations. For example, the results may be influenced by background noise, particularly those based on sound pressure measurements. Practical difficulties occur in cases where reflecting surfaces, or other obstacles, are located close to the source which may prevent access to the required measurement or excitation points. Where the response to a known force is required, the device used to excite the structure, usually a shaker, may affect the structural behaviour and may also radiate noise which can complicate acoustic measurements. Moreover, the effectiveness of force transmission from the shaker to the structure decreases with increasing frequency, especially when a stinger is used. Good discussions on shaker–stinger-structure coupling effects can be found for example in [4,5]. In order to overcome these limitations, a 'reciprocal' technique can be applied; in the proposed method the structure under test is excited by sound in a reverberation chamber and the consequent vibration response is measured, for example using accelerometers.

The idea of using reciprocity for acoustics was proposed by Lord Rayleigh [6] who stated that the acoustic pressure produced at a point *A* in a fluid by a simple harmonic, omni-directional, point-source having a certain volume velocity and located at another point *B* in the fluid is the same as the pressure that would be produced at *B* by the same source located at point *A* [7]. This applies irrespective of the presence of arbitrary boundaries to the fluid.

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The principle of reciprocity can also be applied to structural response, where the force and response positions on a structure can be exchanged.

The principle of acoustic reciprocity was extended by Lyamshev [8,9] to the case of the vibro-acoustic behaviour of an elastic structure, as illustrated in Fig. 1. He demonstrated a relationship between the acoustic pressure at a point *B* due to a vibrating structure subjected to a harmonic mechanical force at a point *A* and its reciprocal situation: that is, in the absence of the mechanical force, the vibration velocity that is produced at the point *A* on the structure due to an acoustic excitation by a point source located in the fluid at the point *B*.

Fahy presented review papers in 1995 [10], and in later in 2003 [11], in which, along with a historical description of the reciprocity principle in vibro-acoustics, several applications are discussed. In contrast to the method outlined in this work, in most of the cases discussed by Fahy the methods do not rely on the use of a reverberation chamber. These papers represent a comprehensive overview of reciprocity in vibro-acoustics and give good background for the method proposed here. An interesting method, named the 'monopole array model', is discussed which consists of an application of reciprocity to the noise radiated from vibrating surfaces. It is based on subdividing the radiating surface into small areas radiating as correlated or uncorrelated monopoles and in an experimental characterisation of the monopole properties through reciprocity. The extension of the reciprocity principle to include the vibrating structure, as demonstrated by Lyamshev, is also discussed in [10] and further applications are detailed by Verheij in [12,13]. In the examples summarised in the above-mentioned papers it is also shown how reciprocity can be of support in developing and applying inverse force identification techniques. In these cases the transfer function between sound pressure at a receiver location and force applied to a structure can be alternatively obtained by placing an omnidirectional source with known volume velocity at the receiver position and by measuring the free velocity thus generated on the structure in the same position and direction as the original excitation force. A similar principle is envisaged in this work although the presence of a reverberant room relaxes the requirements necessary on the source.

The reciprocity principle with 'monopole array model' was applied by Zheng et al. [14] to measure the sound pressure radiated by an internal combustion engine. The engine surface was divided into discrete sub-areas and it was shown that the mean-square pressure at the receiver point could be found from the sound power of each sub-area and the acoustic transfer function between the surface pressure on the engine and the volume velocity of a monopole source. The sound intensity scanning method was employed to measure the sound power of the operating engine and the acoustic transfer function was measured reciprocally between the source strength of a monopole source and the sound pressure measured with a microphone located close to the non-operating engine.

Application of Lyamshev reciprocity has led to various uses in vibro-acoustic problems. It has been adopted for example in studying tyre induced vehicle interior noise [15,16]. Mason and Fahy [17] proposed this technique to measure the transfer function between a point force acting on the exterior of an aircraft fuselage and the sound pressure generated inside the cabin. As the fuselage is a non-uniform structure, a procedure was devised whereby the fuselage is discretised into a number of segments and where the transfer function from the internal volume velocity due to a monopole source and the resulting surface vibration volume velocity on each discrete segment need to be measured. This approach was first validated using a clamped plate on the top of a box excited by a point source from the inside. This technique was then applied by MacMartin et al. [18] to a real aircraft fuselage. Comparison with the direct method was also presented with very good agreement.

In general, Fahy [10] also reports that reciprocal techniques have often been found to be cheaper, less time-consuming, more convenient and sometimes also more accurate than the equivalent direct measurements. However, in terms of characterising sound sources and measuring sound power from operating machinery, international standards are focussed on direct techniques such as the free field method [1], the reverberation room method [2] and the sound intensity method [3]. Other important established direct techniques to measure the acoustic intensity vector field, and therefore to quantify acoustic power, are the Nearfield Acoustic Holography (NAH) [19] and its extension to sound radiated in an enclosed environment, named Phonoscopy [20]. These techniques are largely employed in applications aimed at localising noise sources and they can provide a high degree of accuracy. However, they rely on a large number of sensors and this limits their usage for affordability reasons.

Crocker and Price [21] have shown that Statistical Energy Analysis (SEA) can be successfully used in determining the vibro-acoustic properties of panels. In particular they have shown comparisons between SEA predictions and measurements performed on a panel in a reverberant room and in between two adjacent rooms to measure the radiation resistance and transmission loss of the panel. To measure these quantities a direct technique was adopted while other SEA parameters, such as coupling factors and modal densities, were measured using reciprocity. This technique has also been applied to ribbed panels [22] and to foam-filled honeycomb sandwich panels showing satisfactory results of radiation resistance compared with predicted formulae [23].

This paper focuses on the measurement of the radiation efficiency and radiated sound power of a vibrating structure using a reciprocity technique. In particular, the Lyamshev technique is extended to include measurements in a reverberation chamber. Only the case of mechanical excitation is considered, although it is recognised that the radiation efficiency due to acoustic excitation is sometimes of interest. After presenting the theory and the methodology adopted (Sections 2 and 3) attention is focused on several examples to illustrate the applicability of the method. Firstly a simple rectangular plate with free edges is considered with two different thicknesses. The results are compared with measurements made using a conventional direct approach in

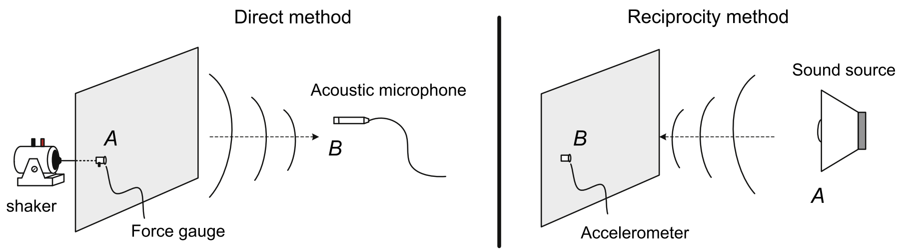

Fig. 1. Illustration of direct and reciprocity techniques. *A*: Exciter location. *B*: Response location.