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Analysis of absorption in situ with a spherical microphone array

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ABSTRACT

Measured values of acoustic absorption often vary between the laboratory and the field due to deficiencies in standard measurement methods. This paper introduces a new method of measuring acoustic absorption in the field using a spherical microphone array. Plane-wave decomposition is used to separate direct energy from reflected energy when the array is placed adjacent to the absorptive sample. Additional signal processing techniques including the Dolph–Chebyshev beampattern and Delay-and-Sum processing are introduced and used to improve the method. The method is verified by simulation for normal and oblique incidence and by experiment for normal incidence.

1. Introduction

Acoustic designers select materials according to their acoustic absorption coefficient in order to achieve a desired reverberation time in acoustically-sensitive rooms or to reduce the sound level in noisy environments. This paper introduces a new method of measuring acoustic absorption in the field using a spherical microphone array. The standardized laboratory procedures for measuring the absorption coefficient use either a reverberation chamber [1] or an impedance tube [2,3]. Discrepancies arise when comparing the results of these methods due to deviations from ideal diffusivity in the reverberation chamber, diffraction at the sample edges in the reverberation chamber, differences in mounting condition that affect the sample frame vibration, and deviations from the local-reaction assumption [4].

Likewise, such factors can cause the absorption coefficient in situ, or on location, to differ from that measured in the laboratory: actual sample size (the "area effect") [5], mounting condition [6], and incident sound field [7]. A number of measurement procedures have been developed to characterize absorption in situ instead of under idealized laboratory conditions. The procedures can generally be divided into wave-field analysis methods and separation methods. Wave-field analysis methods measure direct and reflected components together and use a wave-propagation model in front of the surface to extract the normal surface impedance or directional reflection coefficient [8–10]. More recently a combined particle velocity–pressure sensor has been used for measuring absorption in situ [11]. Separation methods, by contrast, require separating the following components, generally for a single incidence angle:

(i) acoustic pressure from sound wave incident on the sample,
(ii) acoustic pressure from sound wave reflected from sample,
(iii) parasitic reflections that contribute neither to (i) nor (ii).

The reflection coefficient, which leads to the absorption coefficient, is calculated by dividing component (ii) by component (i). A classic separation strategy is to use a time window to isolate components (i) and (ii) [12]. Component (ii) can also be isolated by subtracting component (i) after obtaining it separately in an equivalent free field measurement [13], although the subtraction method is very sensitive to environmental conditions [14]. Array processing has been applied previously to an in situ separation-method technique [15]. Components (i) and (ii) were separated by beamforming with a linear array placed perpendicularly between the source and the surface of interest. However, this array method only permits measurements at normal incidence.

The spherical array method in this paper separates components (i) and (ii) by arrival direction on the array, which is placed adjacent to the sample. Depending on the sample size, time windowing may or may not be required to filter parasitic reflections from the sample edges. This method may be well-suited for use in situ because of the potential to simultaneously isolate multiple components (i) and (ii) from (iii) using plane-wave decomposition. The method described here was proposed in an earlier publication [16] and is further developed and experimentally validated in this paper.

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Section 2 reviews beamforming and other signal processing techniques. Section 3 then describes the application of beamforming to the measurement of acoustic absorption. Simulation studies are presented in Section 4, followed by experimental validations in Section 5. The paper closes with observations and suggestions for future work in Section 6.

2. Signal processing techniques

2.1. Spherical Fourier transform

The spherical Fourier transform converts a spatial function between spatial domain and spherical harmonics domain. Consider a square-integrable function on a sphere, such as acoustic pressure $p(\theta,\phi)$. The spherical Fourier transform is defined as [17]:

$$p_{nm} = \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} p(\theta,\phi) Y_n^{m^*}(\theta,\phi)\sin\theta d\theta d\phi \tag{1}$$

If the coefficients p_{nm} are known, the inverse spherical Fourier transform reconstructs the function in spatial domain [17]:

$$p(\theta,\phi) = \sum_{n=0}^{\infty}\sum_{m=-n}^{n} p_{nm} Y_n^m(\theta,\phi) \tag{2}$$

where the spherical harmonics basis functions are defined as:

$$Y_n^m(\theta,\phi) \equiv \sqrt{\frac{(2n+1)}{4\pi}\frac{(n-m)!}{(n+m)!}} P_n^m(\cos\theta) e^{im\phi} \tag{3}$$

where n is the spherical harmonics order, m is an index running from $-n$ to n, P_n^m is the associated Legendre function, and $i=\sqrt{-1}$. If the function is sampled spatially across a sphere, for example with a spherical array, the integral in Eq. (1) is approximated by a weighted sum at M discrete microphone locations (θ_j,ϕ_j). The correct choice of sampling locations and weights, w_j, can make this approximation exact if p_{nm} is band limited [18]:

$$p_{nm} = \sum_{j=1}^{M} w_j p(\theta_j,\phi_j) Y_n^{m^*}(\theta_j,\phi_j) \tag{4}$$

2.2. Spherical array beamforming

Plane-wave decomposition is the process of extracting the plane-wave components that comprise a sound field. If the sound field on the surface of the sphere is known, then plane-wave decomposition yields the complex amplitude and arrival direction of plane-wave components. The derivation below shows that plane-wave decomposition is a special case of beamforming.

The pressure from a single unit-amplitude plane wave is given by [19]:

$$p(k,r,\theta,\phi,\theta_l,\phi_l) = \sum_{n=0}^{\infty}\sum_{m=-n}^{n} b_n(kr) Y_n^{m^*}(\theta_l,\phi_l) Y_n^m(\theta,\phi) \tag{5}$$

where $b_n(kr)$ for an open sphere of radius r is given by [19]:

$$b_n(kr) = 4\pi i^n j_n(kr) \tag{6}$$

Here, $p(k,r,\theta,\phi,\theta_l,\phi_l)$ is the pressure at point (θ,ϕ) on the sphere due to a plane wave arriving from direction (θ_l,ϕ_l). Also, $k=\omega/c$ is the wavenumber for angular frequency ω and speed of sound c, and j_n is the spherical Bessel function of order n. Taking the spherical Fourier transform in Eq. (1) of Eq. (5) yields:

$$p_{nm}(k,r) = b_n(kr) Y_n^{m^*}(\theta_l,\phi_l) \tag{7}$$

Eq. (7) can be generalized for an infinite number of plane waves by integrating over all arrival directions (θ_l,ϕ_l). The amplitudes of the incident plane waves, expressed as a density function, are given by $a(k,\theta_l,\phi_l)$:

$$\begin{aligned} p_{nm}(k,r) &= \int_{\phi_l=0}^{2\pi}\int_{\theta_l=0}^{\pi} a(k,\theta_l,\phi_l) b_n(kr) Y_n^{m^*}(\theta_l,\phi_l) \sin\theta_l\, d\theta_l\, d\phi_l \\ &= a_{nm}(k) b_n(kr) \end{aligned} \tag{8}$$

Rearranging Eq. (8) to solve for the amplitude–density coefficients yields:

$$a_{nm}(k) = \frac{p_{nm}(k,r)}{b_n(kr)} \tag{9}$$

Applying the inverse spherical Fourier transform in Eq. (2) to the expression in Eq. (9) converts the amplitude-density coefficients, a_{nm}, back to the spatial domain, $a(k,\theta_l,\phi_l)$, as a function only of the pressure coefficients at the surface of the sphere, p_{nm}, and b_n. Generalizing plane-wave decomposition to spherical array processing leads to [18]:

$$y(kr,\theta_l,\phi_l) = \sum_{n=0}^{N}\sum_{m=-n}^{n} d_n \frac{p_{nm}(k,r)}{b_n(kr)} Y_n^m(\theta_l,\phi_l) \tag{10}$$

where an extra term d_n has been added to the right side of the expression. This term is a place-holder for beampattern coefficients used in processing techniques such as the Dolph–Chebyshev beampattern and Delay-and-Sum. When $d_n=1$ and $N\to\infty$, then $y(k,\theta_l,\phi_l) = a(k,\theta_l,\phi_l)$, and the expression yields plane-wave decomposition, which is a special case of beamforming. In Eq. (10), the sum over n is only taken up to order N, the maximum order that can be achieved according to the spatial sampling scheme and number of microphone locations around the sphere. At each frequency of interest, the plane-wave decomposition is performed in two stages. First the pressure coefficients p_{nm} are determined up to order N using Eq. (4). Next, the plane-wave decomposition is computed by Eq. (10) to find the plane-wave amplitude in the desired direction.

2.3. Dolph–Chebyshev beampattern

If one wave is incident from the array look direction (θ_l,ϕ_l), a second wave incident from another direction may still affect the array output $y(k,\theta_l,\phi_l)$. The array output y has contributions from both the former wave and the latter wave. The effect of the secondary wave depends on the beampattern side-lobe level. To reduce the interference from secondary waves, a Dolph–Chebyshev beampattern is introduced. Dolph–Chebyshev beampatterns ensure an optimal minimum main-lobe width for a user-specified side-lobe level. A simulation is conducted and reported in Section 4.2 to determine an appropriate level of side-lobe attenuation. Below is a summary of how to apply Dolph–Chebyshev beampatterns to spherical arrays [20].

For an array of order N, the user specifies the main-lobe to side-lobe amplitude ratio, $1/S$. The parameter x_0 is calculated for determining the null-to-null beamwidth $2\beta_0$. L is the Chebyshev-polynomial order, which is set to $2N$. A relation among S, x_0, and β_0 is given by

$$x_0 = \cosh\left[\frac{1}{L}\cosh^{-1}(S)\right] \tag{11}$$

$$\beta_0 = 2\cos^{-1}\left[\frac{1}{x_0}\cos\left(\frac{\pi}{2L}\right)\right] \tag{12}$$

The amplitude ratio $1/S$ is used to calculate the spherical-array beampattern coefficients, d_k, which are used in Eq. (10).

$$d_k = \frac{2\pi}{S}\sum_{q=0}^{k}\sum_{j=0}^{N}\sum_{m=0}^{j} \frac{1-(-1)^{m+q+1}}{m+q+1}\frac{j!}{m!(j-m)!}\times\left(\frac{1}{2^j}\right) t_{2j}^{2N} p_q^{2N} x_0^{2j} \tag{13}$$