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Disorder in a periodic Helmholtz resonators array

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ABSTRACT

This paper considers the disorder in a periodic duct–resonator system. The transfer matrix method is used to investigate wave propagation in the duct. Two cases are investigated: the disorder in periodic distance and the disorder in the geometries of Helmholtz resonators. Different from the original attenuation characteristic brought about by pure periodic system, it is found that the disorder in the geometries of resonators with the periodic distance being kept unchanged provides a useful way for the design of such a system to achieve a relatively wide noise attenuation band and to track some narrow noise peaks within it.

1. Introduction

A periodic system is composed of a number of identical elements that are joined together end to end and/or side by side to form a whole complex [1]. Owing to the periodicity, the wave propagation in a periodic system exhibits pass-band and stop-band behavior [2]. A periodic system is sometimes imperfect; it may contain defects or perturbations. A single disorder of an infinite periodic system, which can be regarded as two semi-infinite periodic systems connecting through the disordered element, was studied [3]. It was found that defects in the perfect periodicity may lead to narrow frequency transmission bands (i.e. defect states) within the original stop-band gaps [4].

Sometimes the defect means the adiabatic variations of the geometries of some "periodic" elements in the whole system. The perturbations in the geometries of the "periodic" element are random and have some statistical properties. Wave propagation through a medium with random impurity modulation will cause the phenomenon of Anderson localization [5], which was originally discovered in the field of solid state physics and then introduced to the acoustic context [6]. When the random irregularity of the geometries of the "periodic" elements is small compared to its mean value, as a perturbation, this kind of system is sometimes called a near-periodic system [7]. The study of vibration localization due to random disorder in near-periodic structures has been the subject of much recent research [8].

Sometimes the defect means the non-adiabatic variations of the geometries of the "periodic" element, which means that substantial geometric variations occur from one cell to another [2]. The non-adiabatic local perturbation of the geometries affects the global characteristics of the whole system, which is then called a quasi-periodic structure [9]. The quasi-periodic system can be described by the "quasi-Bloch" theory [10]. It has been found that the spectrum of a quasi-periodic structure is a discrete dense set with discontinuous spectral intensities which clearly lie between a periodic and a near-periodic system [10].

This paper considers the imperfect periodic duct–resonator system. The defects contain both the disorder in periodic distance and the disorder in the geometries of Helmholtz resonators. Sometimes the variation of periodic elements is adiabatic, which can be regarded as a near-periodic system. However, sometimes the imperfection in periodicity is man-made and the number of periodic elements is relatively small, which means that the system cannot be adequately described in a statistical way; this paper will look further into this case.

2. Theoretical analysis

As shown in Fig. 1, a "periodic" cell comprises a duct segment with a resonator attached to its left side. In this paper, only the lossless case is considered. When considering the irregularity of the periodic distance between any two nearby resonators and the geometries of Helmholtz resonators, the system can no longer be represented by a single transmission matrix **T** and a single periodic distance D as it is in the pure-periodic case [11,12]. Rather, we

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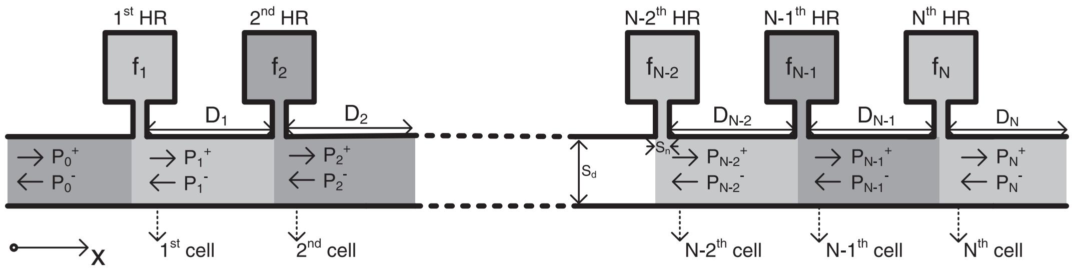

Fig. 1. A duct with N Helmholtz resonators.

should specify each transmission matrix and "periodic" distance, noted as $\mathbf{T}_n$ and D_n for $n = 1, 2,\ldots, N$. Similar to the pure-periodic case, the diameter of the resonator neck is assumed to be negligible compared to the length of the duct segment between two nearby resonators, D_n. The frequency range considered is well below the cut-on frequency of the duct. In the duct segment of the nth cell, the sound traveling in positive- and negative-x directions can be described with sound pressure $P_n^+ = C_n^+ e^{-jk(x_n - D_n)}$ and $P_n^- = C_n^- e^{jk(x_n - D_n)}$ for $0 \le x_n \le D_n$, where C_n^+ and C_n^- are complex constants and k is the wave number. If $\mathbf{c}_n = \begin{bmatrix} C_n^+ & C_n^- \end{bmatrix}^T$ and $\mathbf{c}_{n+1} = \begin{bmatrix} C_{n+1}^+ & C_{n+1}^- \end{bmatrix}^T$ represent the wave components in the duct segment of the nth and $n+1$th elements of the "periodic" system, then it can be related through a wave transfer matrix $\mathbf{T}_{n+1}$ as follows:

$$\mathbf{c}_{n+1} = \mathbf{T}_{n+1}\mathbf{c}_n. \tag{1}$$

As discussed before [11,12], the transfer matrix can be expressed as

$$\mathbf{T}_n = \begin{bmatrix} e^{-jkD_n} & 0 \\ 0 & e^{jkD_n} \end{bmatrix} \begin{bmatrix} 1-\xi_n & -\xi_n \\ \xi_n & 1+\xi_n \end{bmatrix}, \tag{2}$$

where $\xi_n = Z_d/2Z_n$, Z_d/Z_n is the acoustic impedance of the duct/nth resonator respectively. Furthermore, the transfer matrix $\mathbf{T}_n$ can be rewritten in terms of the transmission and reflection coefficients, t_n and r_n, as [13]

$$\mathbf{T}_n = \begin{bmatrix} e^{-jkD_n} & 0 \\ 0 & e^{jkD_n} \end{bmatrix} \begin{bmatrix} 1/t_n^* & -(r_n/t_n)^* \\ -r_n/t_n & 1/t_n \end{bmatrix}, \tag{3}$$

where the superscript * means conjugation. It follows that

$$\mathbf{c}_{n+1}\mathbf{c}_{n+1}^{T*} = \mathbf{T}_{n+1}(\mathbf{c}_n\mathbf{c}_n^{T*})\mathbf{T}_{n+1}^{T*}, \tag{4}$$

where the superscript T means transposition; Eq. (4) can be rewritten in vector form as

$$\mathbf{e}_{n+1} = \mathbf{A}_{n+1}\mathbf{e}_n, \tag{5}$$

where the entries of the vectors $\mathbf{e}_{n+1}$ and $\mathbf{e}_n$ can be expressed in terms of the entries of $\mathbf{c}_{n+1}$ and $\mathbf{c}_n$, and the entries of the 4×4 matrix $\mathbf{A}_{n+1}$ can be expressed in terms of the entries of $\mathbf{T}_{n+1}$ [13], as

$$\mathbf{e}_n = \begin{bmatrix} C_n^+C_n^{+*} & C_n^+C_n^{-*} & C_n^-C_n^{+*} & C_n^-C_n^{-*} \end{bmatrix}^T, \tag{6}$$

and

$$\mathbf{A}_{n+1} = \begin{bmatrix} 1/|t_{n+1}|^2 & -r_{n+1}/|t_{n+1}|^2 & -r_{n+1}^*/|t_{n+1}|^2 & |r_{n+1}|^2/|t_{n+1}|^2 \\ -r_{n+1}^*\delta_{n+1}/t_{n+1}^{*2} & \delta_{n+1}/t_{n+1}^{*2} & r_{n+1}^{*2}\delta_{n+1}/t_{n+1}^{*2} & -r_{n+1}^*\delta_{n+1}/t_{n+1}^{*2} \\ -r_{n+1}/\delta_{n+1}t_{n+1}^2 & r_{n+1}^2/\delta_{n+1}t_{n+1}^2 & 1/\delta_{n+1}t_{n+1}^2 & -r_{n+1}/\delta_{n+1}t_{n+1}^2 \\ |r_{n+1}|^2/|t_{n+1}|^2 & -r_{n+1}/|t_{n+1}|^2 & -r_{n+1}^*/|t_{n+1}|^2 & 1/|t_{n+1}|^2 \end{bmatrix}, \tag{7}$$

where $\delta_{n+1} = \exp(-2jkD_{n+1})$. It should be noted that the final diagonal entry of the matrix $\mathbf{A}_{n+1}$ has the value $\mathbf{A}_{n+1}(4,4) = 1/|t_{n+1}|^2$ [9], and thus knowledge of $\mathbf{A}_{n+1}$ leads immediately to the modulus squared transmission coefficient between the nth and $n+1$th "periodic" elements. Furthermore, for the whole system, there is

$$\mathbf{e}_N = \left(\prod_{n=1}^{N}\mathbf{A}_{N+1-n}\right)\mathbf{e}_0 = \mathbf{\Lambda}\mathbf{e}_0, \tag{8}$$

where $\mathbf{A}_n$ is the matrix derived from $\mathbf{T}_n$ and $\mathbf{\Lambda}$ is the corresponding matrix of the whole system.

2.1. Random disorder

If the periodic system subject to random disorder and the random variations in the properties of each "periodic" element are statistically independent, then Eq. (8) can be described by the ensemble average behavior of the system as [9]

$$E[\mathbf{\Lambda}] = \prod_{n=1}^{N}E[\mathbf{A}_{N+1-n}]. \tag{9}$$

Here, $E[\ ]$ represents the ensemble average. If the various "periodic" elements have the same probability distribution, then Eq. (9) can be reduced to

$$E[\mathbf{\Lambda}] = E[\mathbf{A}]^N. \tag{10}$$

If the duct–resonator system is initially periodic, with periodic distance D and the transmission and reflection coefficients of a side-branched Helmholtz resonator t and r, the matrices $\mathbf{A}_{n+1}$ are all the same with the omission of the subscript $n+1$ in Eq. (7). If the Helmholtz resonators are identical while the periodic distance D can be described by a Gaussian distribution with the probability density function $p(D)$, the parameters δ in Eq. (7) can be expressed as

$$\delta = \int_{-\infty}^{\infty} e^{-2jkD}p(D)dD = e^{-2jkD_0 - 4\sigma^2k^2/2}. \tag{11}$$

where D_0 is the mean "periodic" distance and σ is the standard deviation.

Eq. (10) can also be expressed in the form

$$E[\mathbf{\Lambda}] = \mathbf{P}\Gamma^N\mathbf{P}^{-1}, \tag{12}$$

where $\mathbf{\Gamma}$ is a diagonal matrix containing the eigenvalues of $E[\mathbf{A}]$ and $\mathbf{P}$ is a matrix whose columns are the corresponding eigenvectors. If N is allowed to tend to infinity and $E[\mathbf{\Lambda}]$ has at least one real eigenvalue greater than unity, there is

$$\lim_{N\to\infty}\left\{E\left[1/|t_N|^2\right]\right\} = N\ln\lambda_{\max}, \tag{13}$$

where t_N is the transmission loss of the whole "periodic" duct–resonator system.

2.2. Man-made disorder

However, sometimes the disorder is not an imperfection but a man-made disorder to achieve a modified filter characteristic of