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Vibration response of stepped FGM beams with elastically end constraints using differential transformation method

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ABSTRACT

In this paper, free vibration response of stepped beams made from functionally graded materials (FGMs) is investigated. The beams are supported by various types of elastically end constraints. The differential transformation method (DTM) is employed to solve the governing differential equations of such beams in order to obtain natural frequencies and mode shapes. The power law distribution is used and modified to describe material compositions across the thickness of the beams made of FGMs. Two main types of the stepped FGM beams in which their material compositions can be described by using the modified power law distribution are selected to investigate their vibration behaviour. The significant parametric studies such as step ratio, step location, boundary conditions, spring constants and material volume fraction are taken into investigation.

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1. Introduction

FGMs are the new class of composite materials which have spatially varying material properties. The earliest FGMs were introduced by Japanese scientists in the mid-1980s as ultra-high temperature-resistant materials for aerospace applications. Recently, these materials have found other uses in electrical devices, energy transformation, biomedical engineering, optics, etc. [1]. A beam made from FGM is considered as an important structural element which needs to be well-designed when subjected to dynamic loading. The FGM beams are shaped to step configuration for using in engineering structures due to fabrication, assembly and space constraints. At the end supports of the beams, damaged or imperfect supports are modelled using elastically end constraints which compose of translational and rotational springs [2]. For example, in case of imperfectly clamped condition, this can be simulated by using moderate or hard stiffness of translational and rotational springs. Therefore, it is necessary to consider vibration behaviour of stepped FGM beams with elastically end constraints.

In the study of Wattanasakulpong et al. [3], a multi-step sequential infiltration technique was used to fabricate FGM beams, for vibration testing. An elaborate discussion on FGM fabrication, microstructure and material volume fraction analysis as well as vibration experimental setup was presented in the study. Kapuria et al. [4,5] manufactured ceramic–metal FGM beams via powder metallurgy and thermal spray techniques, for bending and vibration testings. Based on the open literature, many researchers have concentrated their attention on investigating free vibration response of uniform FGM beams. For example, Sina et al. [6] presented the modified first order shear deformation theory (FSDT) to deal with vibration problem of the uniform FGM beams. Simsek [7] used the Lagrange multiplier method to obtain fundamental frequencies of FGM beams, using different beam theories. The vibration analysis of FGM beams having cracks at the edge was investigated and presented in Refs. [8,9]. An improved third order shear deformation theory (TSDT) was applied to analyse thermal buckling and elastic vibration of FGM beams using the Ritz method, Wattanasakulpong et al. [10,11]. On the basic of Euler–Bernoulli beam theory, nonlinear governing differential equation was constructed by Fallah and Aghdam [12] in order to deal with the problems of thermo-mechanical buckling and vibration analyses of FGM beams resting on elastic foundation. To obtain the analytical solutions of the nonlinear governing equation, He's variational method was employed to find out the results of critical bucking temperatures and natural frequencies of the beams.

On the investigations of vibration of stepped beams, there were a number of reports dealing with the topic [13–16]. However, the stepped beams selected to investigate free vibration characteristics in the past are made from isotropic and laminated composite materials, using different theories and methodologies. To consider vibration analysis of beams supported by elastically end constraints, Lai et al. [17] used an Adomain decomposition method to solve free vibration problem of Euler–Bernoulli beams with several types of elastic boundary conditions.

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The DTM is an effective mathematical tool that can be applied to solve ordinary and partial differential equations. By using the DTM to solve vibration problem, fundamental and higher frequencies as well as their corresponding mode shapes can be obtained accurately without any frequency missing. Malik and Dang [18] had successfully implemented the DTM to deal with vibration analysis of isotropic beams supported by several end conditions. Other researchers [19–21] applied the DTM to solve many cases of vibration problems of beams. Pradhan and Reddy [22] also used the DTM to find out the buckling results of a single walled carbon nanotube.

According to the literature survey presented above, it is found that most researchers have analysed and presented only the vibration of uniform FGM beams with common boundary conditions. Therefore, free vibration analysis of stepped FGM beams supported by elastically end constraints is chosen to investigate their vibration behaviour in this study. The DTM is applied to solve the governing equations of the stepped FGM beams in order to obtain accurate frequency results and mode shapes. Several important aspects such as step ratio, step location, boundary conditions, spring constants as well as the material volume fraction index which have impacts on natural frequencies of such beams are investigated and discussed in detail.

2. Stepped FGM beams

Two types of stepped FGM beams made from ceramic–metal phases are chosen to investigate their vibration behaviour. The geometries and descriptions of the beam types are shown in Fig. 1. The beams shown in this figure are supported by elastic conditions at both ends including translational and rotational springs which are defined as the E-E beams.

It is known that FGMs are inhomogeneous spatial composite materials, typically composed of a ceramic–metal pair of materials. The material compositions are varied throughout the thickness direction from the top surface (ceramic 100%–metal 0%) to the bottom one (ceramic 0%–metal 100%). It can be seen in Fig. 1 that the material compositions at point p_1 and p_2 are the same for the FGM Type-I. For example, assuming that the material compositions at p_1 are (ceramic 70%–metal 30%) and those of p_2 are (ceramic 70%–metal 30%) too. However, for FGM Type-II, the material compositions at p_1 consist of the mixture of ceramic and metal (ceramic 70%–metal 30%) whereas the material at p_2 is pure ceramic (ceramic 100%–metal 0%). In terms of fabrication, FGM Type-I can be easier produced from grinding or machining uniform FGM beam to required size of stepped beam. While, FGM Type-II may require a special technique of powder metallurgy processing with the aid of stepped mount. However, FGM Type-II is more suitable to be used in moisture and thermal environment because it has more percentage of ceramic that can resist corrosion and thermal stress for metal phases.

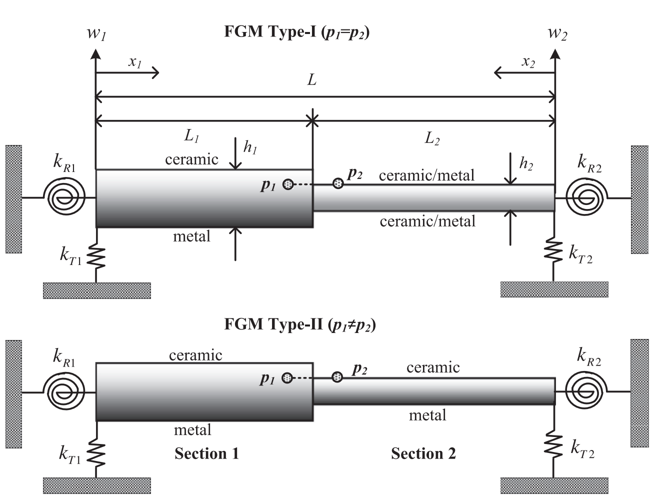

Fig. 1. Geometries and descriptions of two types of stepped FGM beams.

Table 1
The volume fractions of ceramic based on the power law distribution of the stepped FGM beams.

	FGM Type-I	FGM Type-II
Section 1	$V_{c1} = \left(\frac{z}{h}+\frac{1}{2}\right)^N;\quad z \in \left[\frac{h}{2}, \frac{-h}{2}\right]$	$V_{c1} = \left(\frac{z}{h}+\frac{1}{2}\right)^N;\quad z \in \left[\frac{h}{2}, \frac{-h}{2}\right]$
Section 2	$V_{c2} = \left(\frac{z}{h}+\frac{1}{2}\right)^n;\quad z \in \left[\frac{\xi h}{2}, \frac{-\xi h}{2}\right]$	$V_{c2} = \left(\frac{z}{\xi h}+\frac{1}{2}\right)^n;\quad z \in \left[\frac{\xi h}{2}, \frac{-\xi h}{2}\right]$

It is defined that the thickness of the stepped beams at Section 1 is given by $h_1 = h$ and at Section 2 is $h_2 = \xi h$ in which ξ is the step ratio parameter ($0 < \xi \leqslant 1$).

Based on the rule of mixture, the effective material properties ($\overline{P}_j$); such as Young's modulus (E) and mass density (ρ), can be written as:

$$\overline{P}_j = \overline{P}_{mj} V_{mj} + \overline{P}_{cj} V_{cj} \quad (j = 1, 2), \tag{1}$$

where the subscript $j = 1$ and 2 denote respectively the Sections 1 and 2 of the stepped FGM beams. $\overline{P}_{mj}, \overline{P}_{cj}, V_{mj}$ and V_{cj} are material properties and the volume fraction of the metal and ceramic corresponding to each beam section, respectively, with the relation

$$V_{mj} + V_{cj} = 1 \quad (j = 1, 2). \tag{2}$$

According to the power law distribution, the volume fraction of ceramic (V_{cj}) in each section of the stepped beams is presented in Table 1.

It is noted that the positive number, $0 \leqslant N, n \leqslant \infty$, is the power law or volume fraction index. The FGM beam becomes a fully ceramic beam when $N = n$ are set to zero. From the above relationship, the material properties in terms of Young's modulus and mass density in each section are expressed as:

$$E_j(z) = (E_{cj} - E_{mj}) V_{cj} + E_{mj}, \quad (j = 1, 2), \tag{3}$$

$$\rho_j(z) = (\rho_{cj} - \rho_{mj}) V_{cj} + \rho_{mj}, \quad (j = 1, 2). \tag{4}$$

Poisson's ratio (ν) is assumed to be a constant value.

To investigate vibration analysis of the stepped FGM beams, the material stiffness components are obtained from $(A_{11}, B_{11}, D_{11})_j = \int \frac{E_j(z)}{[1-\nu^2]}(1, z, z^2)dz$ and the moment of inertia can be calculated by $I_{0j} = \int \rho_j(z)dz$. The upper and lower limits of the integrals are set according to the thickness of considered beam section.

3. Application of DTM to vibration of stepped FGM beams

Consider a classical beam theory (CBT), the partial differential equation used to describe the free vibration in each section of the stepped FGM beams can be expressed as:

$$\frac{\partial^4 w_j(x_j, t)}{\partial x_j^4} + \frac{I_{0j}}{\lambda_j} \frac{\partial^2 w_j(x_j, t)}{\partial t^2} = 0; \quad x_j \in [0, L] \quad (j = 1, 2). \tag{5}$$

It is defined that I_{0j} is the moment of inertia, $\lambda_j = \left(D_{11} - \frac{B_{11}^2}{A_{11}}\right)_j$ is the material stiffness coefficient in each beam section. For harmonic vibration, $w_j(x_j,t) = W_j(x_j)e^{i\omega t}$ is substituted into Eq. (5) to obtain a time independent governing equation as follows.

$$\frac{d^4 W_j(x_j)}{dx_j^4} - \frac{I_{0j}}{\lambda_j} \omega^2 W_j(x_j) = 0, \tag{6}$$

where ω is a natural frequency.

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