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Technical Note

On the adjustment of Helmholtz resonators

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ABSTRACT

A Helmholtz resonator is placed in a room with distinct acoustic modes, and is tuned to one of the corresponding resonant frequencies. The optimal resonator damping ratio is investigated, as a goal-dependent value. For example, minimizing reverberation time requires a different damping ratio from minimizing the sound pressure level. The optimum damping values for a Helmholtz resonator are analytically computed, and then verified by means of experimentation. Furthermore, a construction is introduced which allows for a fine adjustable setting for the eigenfrequency and the damping ratio of the resonator.

1. Introduction

Resonant acoustic modes prevent small and medium-sized rooms from having a smooth frequency response, and also cause long reverberation times. Especially in sound studios or acoustic measurement rooms these effects must be prevented. In order to diminish these effects a Helmholtz resonator (see [1,2]) can be used.

An appreciable smoothing of the frequency response is only possible if the Helmholtz resonator is properly tuned, placed in the right place and the damping of the resonator is set to an appropriate value. To affiliate the optimal parameters it is necessary to understand the physical interaction between a Helmholtz resonator and the room mode. For this we use a model, published by [3], in which the room is modeled as a continuum and the resonator as a simple mass oscillator with a single degree of freedom. The eigenfrequency of a Helmholtz resonator is considered in different sources [4–6] and is not subject of this study. Using this model it is possible to draw an analogy to dynamic vibration absorbers [7], such that it is possible to use similar mathematical approaches.

Up to now the damping ratio of Helmholtz resonators has been designed by trial and error. As a result of this study we achieve optimal damping parameters, which will simplify the application of Helmholtz resonators.

2. Modelling

The free interaction between a Helmholtz resonator and an acoustic mode in a room is described by [3]:

$$\ddot{T}_{\mathrm{N}}(t) + 2\zeta_{\mathrm{N}}\omega_{\mathrm{N}}\dot{T}_{\mathrm{N}}(t) + \omega_{\mathrm{N}}^2 T_{\mathrm{N}}(t) = -\frac{c^2 R_{\mathrm{N}}(\boldsymbol{r}_{\mathrm{HR}})A}{V\Lambda_{\mathrm{N}}}\dot{\xi}(t), \quad (1)$$

$$\ddot{\xi}(t) + 2\zeta_{\mathrm{HR}}\omega_{\mathrm{HR}}\dot{\xi}(t) + \omega_{\mathrm{HR}}^2\xi(t) = \frac{R_{\mathrm{N}}(\boldsymbol{r}_{\mathrm{HR}})}{l_{\mathrm{eff}}}\dot{T}_{\mathrm{N}}(t), \quad (2)$$

in which $T_{\mathrm{N}}(t)R_{\mathrm{N}}(\boldsymbol{r})$ is the potential function of the sound particle velocity of a distinct room mode, with index N. The subscript HR indicates parameters of the Helmholtz resonator. The term ω denotes the eigenfrequency and ζ is the damping ratio. The volume of the room is V, and the mode normalization factor is Λ_{N}. The term c is used for the sound velocity, and A for the orifice area of the resonator. The coordinate of the oscillating air in the resonator orifice is ξ, and the effective length of this air column is l_{eff}. The position of the resonator is given by $\boldsymbol{r}_{\mathrm{HR}}$. Additionally, an active sound source, such as a loudspeaker, can be placed at the position $\boldsymbol{r}_{\mathrm{LS}}$ in the room. In analogy to the resonator, which works as a reactive sound source, the loudspeaker can be modelled by the term $-\frac{c^2 R_{\mathrm{N}}(\boldsymbol{r}_{\mathrm{LS}})}{V\Lambda_{\mathrm{N}}}Q_{\mathrm{LS}}(t)$, in which $Q_{\mathrm{LS}}(t)$ describes the volume per unit time displaced by the loudspeaker membrane, which yields

$$\ddot{T}_{\mathrm{N}}(t) + 2\zeta_{\mathrm{N}}\omega_{\mathrm{N}}\dot{T}_{\mathrm{N}}(t) + \omega_{\mathrm{N}}^2 T_{\mathrm{N}}(t) = -\frac{c^2 R_{\mathrm{N}}(\boldsymbol{r}_{\mathrm{HR}})A}{V\Lambda_{\mathrm{N}}}\dot{\xi}(t) - \frac{c^2 R_{\mathrm{N}}(\boldsymbol{r}_{\mathrm{LS}})}{V\Lambda_{\mathrm{N}}}Q_{\mathrm{LS}}(t). \quad (3)$$

The following arithmetic is analogous to that used for mechanical vibration absorbers in [7]. Concentrating the variables $T_{\mathrm{N}}(t)$ and $\xi(t)$ in a vector $\boldsymbol{x}(t) = \begin{pmatrix} T_{\mathrm{N}}(t) \\ \xi(t) \end{pmatrix}$, Eqs. (2) and (3) can be written as:

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$$\boldsymbol{M}\ddot{\boldsymbol{x}}(t) + \boldsymbol{D}\dot{\boldsymbol{x}}(t) + \boldsymbol{K}\boldsymbol{x}(t) = \boldsymbol{q}(t), \tag{4}$$

with

$$\boldsymbol{M} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \boldsymbol{D} = \begin{pmatrix} 2\zeta_N\omega_N & c_1 \\ c_2 & 2\zeta_{HR}\omega_{HR} \end{pmatrix}, \quad \boldsymbol{K} = \begin{pmatrix} \omega_N^2 & 0 \\ 0 & \omega_{HR}^2 \end{pmatrix}, \tag{5}$$

and

$$\boldsymbol{q}(t) = \begin{pmatrix} -\frac{c^2 R_N(\boldsymbol{r}_{LS})}{V\Lambda_N} Q_{LS}(t) \\ 0 \end{pmatrix}. \tag{6}$$

In the matrix $\boldsymbol{D}$,

$$c_1 = \frac{c^2 R_N(\boldsymbol{r}_{HR})A}{V\Lambda_N} \text{ and } c_2 = -\frac{R_N(\boldsymbol{r}_{HR})}{l_{eff}} \tag{7}$$

are used. In the next step, the Eq. (4) is operated on by the Laplace transform:

$$s^2\boldsymbol{M}\boldsymbol{x}(s) + s\boldsymbol{D}\boldsymbol{x}(s) + \boldsymbol{K}\boldsymbol{x}(s) = \boldsymbol{q}(s) \tag{8}$$

The shorthand

$$\boldsymbol{S}(s) = s^2\boldsymbol{M} + s\boldsymbol{D} + \boldsymbol{K} \tag{9}$$

leads to

$$\boldsymbol{S}(s)\boldsymbol{x}(s) = \boldsymbol{q}(s), \tag{10}$$

$$\boldsymbol{S}^{-1}(s)\boldsymbol{q}(s) = \boldsymbol{x}(s). \tag{11}$$

Multiplying by s yields

$$s\boldsymbol{S}^{-1}(s)\boldsymbol{q}(s) = s\boldsymbol{x}(s), \tag{12}$$

and using the shorthand

$$\boldsymbol{Y}(s) := s\boldsymbol{S}^{-1}(s), \tag{13}$$

results in

$$\boldsymbol{Y}(s)\boldsymbol{q}(s) = s\boldsymbol{x}(s). \tag{14}$$

The sound pressure $p(\boldsymbol{r}_M, s)$ in the room at a point $\boldsymbol{r}_M$ is given by

$$p(\boldsymbol{r}_M, s) = -\rho_0 R(\boldsymbol{r}_M)\dot{T}_N(s), \tag{15}$$

$$= -\rho_0 R(\boldsymbol{r}_M)s\boldsymbol{x}_{11}(s). \tag{16}$$

This means that $\boldsymbol{Y}_{11}(s)$, multiplied by the time invariant factor, $-\rho_0 R(\boldsymbol{r}_M)$, is the transfer function between a source signal, $q(s)$, and the sound pressure, $p(\boldsymbol{r}_M, s)$, at any point in the room, whose location is given by the vector $\boldsymbol{r}_M$. Inserting all of these described values into Eq. (13) yields

$$\boldsymbol{Y}_{11}(s) = \frac{s(s^2 + 2s\zeta_{HR}\omega_{HR} + \omega_{HR}^2)}{-c_1c_2s^2 + (s^2 + 2s\zeta_N\omega_N + \omega_N^2)(s^2 + 2s\zeta_{HR}\omega_{HR} + \omega_{HR}^2)}. \tag{17}$$

Eq. (7), combined with the expression for the eigenfrequency of the Helmholtz resonator,

$$f_0 = \frac{c}{2\pi}\sqrt{\frac{A}{V_{HR}l_{eff}}}, \tag{18}$$

leads to

$$\begin{aligned} c_1c_2 &= -\frac{c^2AR_N^2(\boldsymbol{r}_{HR})}{l_{eff}V\Lambda_N}, \\ &= -\frac{c^2AV_{HR}R_N^2(\boldsymbol{r}_{HR})}{V_{HR}l_{eff}V\Lambda_N}, \\ &= -\omega_{HR}^2\frac{V_{HR}R_N^2(\boldsymbol{r}_{HR})}{V\Lambda_N}. \end{aligned} \tag{19}$$

The term $\frac{V_{HR}R_N^2(\boldsymbol{r}_{HR})}{V\Lambda_N}$ in Eq. (19), is analogous to what [3] termed the coupling parameter ε:

$$\varepsilon = \frac{V_{HR}R_N^2(\boldsymbol{r}_{HR})}{V\Lambda_N}. \tag{20}$$

The term $R_N(\boldsymbol{r})$ is the location-dependent factor of the potential function of the particular velocity, which is in zero in a node and equal to one in an antinode. The term Λ_N can be determined by

$$\Lambda_N = \frac{\iiint_V R(\boldsymbol{r})^2 dr_1 dr_2 dr_3}{V}. \tag{21}$$

In rectangular shaped rooms it is $\Lambda_N = \frac{1}{2}$ for axial modes, $\Lambda_N = \frac{1}{4}$ for tangential modes and $\Lambda_N = \frac{1}{8}$ for oblique modes.

It is useful to cast these equations using dimensionless parameters. In addition to ζ_{HR}, ζ_N and ε, the dimensionless term

$$v = \frac{\omega_{HR}}{\omega_N} \tag{22}$$

is introduced. Using these normalized values, the Eq. (17) can be written as:

$$\boldsymbol{Y}_{11}(s) = \frac{a_1}{b_1}, \tag{23}$$

with

$$a_1 = s(s^2 + 2s\zeta_{HR}v\omega_N + v^2\omega_N^2)$$

and

$$a_2 = s^2\varepsilon v^2\omega_N^2 + (s^2 + 2s\zeta_N\omega_N + \omega_N^2)(s^2 + 2s\zeta_{HR}v\omega_N + v^2\omega_N^2).$$

In the following the system is considered in a steady state, thus the system can be transformed via a Fourier transformation into a frequency spectrum with the variable ω. Further the dimensionless frequency η is used for the term $\frac{\omega}{\omega_N}$. Hence the square norm $|\boldsymbol{Y}_{11}|^2(\eta)$ becomes:

$$|\boldsymbol{Y}_{11}|^2(\eta) = \frac{a_2}{b_2}, \tag{24}$$

with

$$a_2 = \eta^2(\eta^4 + 2(-1 + 2\zeta_{HR}^2)\eta^2v^2 + v^4)$$

and

$$\begin{aligned} b_2 = \Big(&\eta^8 + v^4 - 2\eta^2v^2\Big(1 - 2\zeta_{HR}^2 + \Big(1 + \varepsilon - 2\zeta_N^2\Big)v^2\Big) \\ &-2\eta^6\Big(1 - 2\zeta_N^2 + \Big(1 + \varepsilon - 2\zeta_{HR}^2\Big)v^2\Big) + \eta^4\Big(1 + 2\Big(\varepsilon \\ &+2\Big(-1 + 2\zeta_N^2\Big)\Big(-1 + 2\zeta_{HR}^2\Big)\Big)v^2 + 8\varepsilon\zeta_N\zeta_{HR}v^3 + \Big(1 + \varepsilon\Big)^2v^4\Big)\Big)\omega_N^2. \end{aligned}$$

Clearly ω_N affects $|\boldsymbol{Y}_{11}|^2(\eta)$ with the factor $\frac{1}{\omega_N^2}$. When $|\boldsymbol{Y}_{11}|^2(\eta)$ is plotted logarithmically, a change of ω_N only causes a translation but not a change of the curve's form, so that ω_N can be set to an arbitrary value. For ease of comparison, ω_N should be chosen such that the function with the parameter $\varepsilon = 0$ has its maximum at 1 =: 0 dB. This function with the parameter $\varepsilon = 0$ describes the square of the sound pressure in the room without a Helmholtz resonator in it. Solving the equation for ω_N,

$$|\boldsymbol{Y}_{11}(\varepsilon = 0, v = 1, \eta = 1)|^2 = 1 \tag{25}$$

yields $\omega_N = \frac{1}{2\zeta_N}$. Inserting this into Eq. (24) leads to

$$|\boldsymbol{Y}_{11}|^2(\eta) = \frac{a_3}{b_3} \tag{26}$$

with

$$a_3 = 4\zeta_N^2\eta^2(\eta^4 + 2(-1 + 2\zeta_{HR}^2)\eta^2v^2 + v^4)$$