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Gear transmission dynamic: Effects of tooth profile deviations and support flexibility

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ABSTRACT

In this work a non-linear dynamic model of spur gear transmissions previously developed by the authors is extended to include both desired (relief) and undesired (manufacture errors) deviations in the tooth profile. The model uses a hybrid method for the calculation of meshing forces, which combines *FE* analysis and analytical formulation, so that it enables a very straightforward implementation of the tooth profile deviations. The model approach handles well non-linearity due to the variable meshing stiffness and the clearances involved in gear dynamics, also including the same phenomena linked to bearings. In order to assess the ability of the model to simulate the impact of the deviations on the transmission dynamics, an example is presented including profile deviations under different values of transmitted torque. Several results of this example implementation are presented, showing the model's effectiveness.

1. Introduction

Today, gear transmissions are under great pressure to improve their performance, in terms of levels of power, speed, efficiency and compactness. A significant increase in the operating speeds is expected in the medium and long term, and consequently dynamic phenomena will become more important in the future, justifying further interest in the development of more accurate dynamic models.

In gear dynamics there is a particular feature that governs the vibratory behaviour, namely the presence of a parametric excitation as a consequence of the changes in the number of teeth pairs contacting simultaneously. This aspect makes the development of dynamic models cumbersome, because a balance must be achieved between accuracy and computational time [1]. Moreover, due to the non-linearity inherent to contact problems, as well as to clearances and deflections of teeth and supports, the amplitude of the torque also affects the meshing stiffness.

Bearings and gears present a similar behaviour, in the sense that bearings also undergo a parametric excitation, in this case due not only to the changes in the number of rolling elements supporting the transmitted load, but also because of the non-linearity related to clearances and surface contacts.

Therefore, gear transmissions should be considered as a whole, including the dynamic effects of gear and bearings, particularly if a better understanding of the transmission behaviour is required for condition monitoring purposes. With this objective, in previous works the authors presented a numerical model which combined gears and bearings, with the capability of representing all the features described above. In [2] the authors described the model for calculation of meshing and bearing forces, carrying out several quasi-static analyses to show the differences in gear centre orbits, transmission error and meshing stiffness values for several transmitted torques. Subsequently, in [3] the procedure used for gear force calculation, based on a hybrid approach combining numerical and analytical tools, was extended including dissipative forces due to friction and squeeze damping. The model was assessed in dynamic simulations, speeding up the computation time by using a pre-calculated value for the meshing stiffness. Later, in [4] the dynamic model was linearized for several torque levels, obtaining the natural frequencies and mode shapes which are essential to understand the vibratory behaviour of the transmission. Moreover, gear defects such as pitting and cracks were also included, carrying out quasi-static analysis to assess the consequences [5].

Another important kind of deviations should be taken into account when modelling gear transmissions. Although the theoretical form of the profile of the flank of a spur gear is an involute, in practice it is not possible to make perfect profiles, so the real flanks present deviations from the ideal shape. These errors are directly related to the level of noise transmission produced, and have been considered by different authors. Kahraman and Singh [6] classified them as an internal source of excitation which, combined with pitch errors, teeth and supports deflections, gives the so-called transmission error. This error is defined as the difference between the actual angular position of the driven gear and the theoretical position where it would be if the gears were perfectly shaped and infinitely rigid. It is well known that the noise level

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of a transmission is strongly influenced by the manufacturing quality of the gears. This aspect has been studied by Bonori et al. [7] who developed a procedure for generating random profile errors in a range of tolerances established for all pinion and gear teeth. These individual errors were combined for a complete transmission cycle and expanded in a Fourier series for its implementation in a dynamic model. From the simulation results, the authors concluded that the inclusion of these errors leads to a significant increase in the vibration amplitude throughout the whole frequency range. They also pointed out that this increase is most apparent at low speeds and torque loads, where non-linearity related with contact loss can appear more easily. This fact has also been addressed specifically by Ottewill et al. [8], who concluded that even tiny tooth profile errors can have a major effect on gear rattle.

Sometimes the profile changes are desired and they are introduced in a premeditated manner. These consist in modifications of the original profile by certain relief in the near-tip area (tip relief) or at the base of the tooth (bottom relief) with which a significant improvement in the noise and vibration levels is achieved. This is an aspect that has also received attention from many authors. The models that can be found in the literature addressing this kind of modification, generally consider that changes in the profiles are so small that the Line of Action (*LOA*) of the contact forces does not change, in such a way that there is only a reduction in the magnitude of the geometric overlap between the profiles of the teeth in contact, in comparison with the non-modified case [1,9].

In this work, profile deviations such as tip reliefs and undesired profile errors are added to the dynamic model previously developed by the authors, with the aim of analysing the consequences on the dynamic behaviour under several torque loads. As a consequence of the supports flexibility, the amplitude of the transmitted torque modifies the distance between centres and the pressure angle, producing an alteration of the conditions of contact between gears. In addition, due to teeth deformations, both the effective contact ratio and the meshing stiffness are modified. The model presented enables the consideration of these phenomena due to the formulation of the local contact (which is non-linear). Furthermore, the model also takes into account the consequences of teeth deflection as well as the possible changes in the distance between gear centres. This aspect becomes particularly interesting to improve the design procedure to determine the parameters which define the profile relief.

2. Description of the model

In this section the proposed model is briefly explained, paying greater attention to the formulation used to include profile deviations. More details about the model can be found in Refs. [2–5].

Gear forces are obtained following the proposal of Vedmar and Andersson [10] in which the deformation at each gear contact point is formulated as a combination of a global (or structural) term obtained by means of a *FE* model, and a local term described by an analytical approach which derives from Hertzian contact theory.

The tooth profile geometry necessary to build the *FE* model is generated using a rack-type tool following Litvin's vector approach [11]. For the *FE* model it is assumed that the nodes in the inner circle are fixed, that is, where the gears are fitted to the shaft. Multiple load cases are considered, each of which is defined by a unit load perpendicular to the tooth profile located at different radial positions from the root to the tip. Then the *FE* model built for each gear is solved once, before the integration of dynamic equations, obtaining the displacement (flexibility) of the node *j* due to a unitary load applied in the node *i* of the loaded active flank. These flexibilities are used to solve the contact problem imposing the compatibility of geometrical separations (δ_j) and elastic deflections (u_{Tj}) submitted to the complementary condition (in order to avoid non-realistic negative loads) arriving at the following non-linear system of equations for *n* contacting points

$$\begin{aligned} &\delta_j(\{\vec{r}_p, \theta_p\}, \{\vec{r}_w, \theta_w\}) = u_{Tj}(\{\vec{r}_p, \theta_p\}\{\vec{r}_w, \theta_w\}\{F\}) \\ &\textit{submitted to} \quad F_i \geqslant 0; \quad i,j = 1, \ldots, n \end{aligned} \tag{1}$$

where $\{F\}$ is the unknown vector, which contains the contact forces F_i for each active contacting point. Subscripts *p* and *w* refers to the pinion and wheel respectively, while *r* and θ represent the centre and angular position of the gears. Meshing forces are extended including Coulomb friction with a smoothed function to avoid the singularity corresponding to the pitch point contact. Furthermore, meshing damping is formulated as a function of the squeeze film (see [3]).

The elastic deflections (u_{Tj}) in (1) are obtained by addition of the global and local terms for both gears. At the same time, geometrical separations are obtained taking advantage of the analytical properties of involute profiles and rounding arcs (which are introduced at the tooth tip to handle the possibility of corner contacts). Therefore, two different types of contact are considered: involute–involute and involute–tip rounding contact. In the first case, the normal contact force is parallel to the *LOA* whereas for the second scenario the resultant force acts Out of the Line of Action (*OLOA*). This feature provides a smoother transition on the shape of the meshing stiffness.

As it happens with gears, the changing number of bearing rolling elements supporting the load implies a parametric excitation, function of the shaft rotational angle. This time, bearing clearance interacts with the magnitude of the load to be transmitted, defining the angular positions in which the number of rolling elements supporting the load changes. To consider these facts, bearing forces have been formulated following the model proposed by Fukata et al. [12].

The presented gear and bearing formulations are implemented in a dynamic model of a single-stage transmission, which is shown in Fig. 1 as a block diagram. Shaft torsional and flexural deflections are taken into account by spring-damper elements, while non-linear forces of gears and bearings are represented by two-way arrows. A reference framework is defined with *z-axis* along the

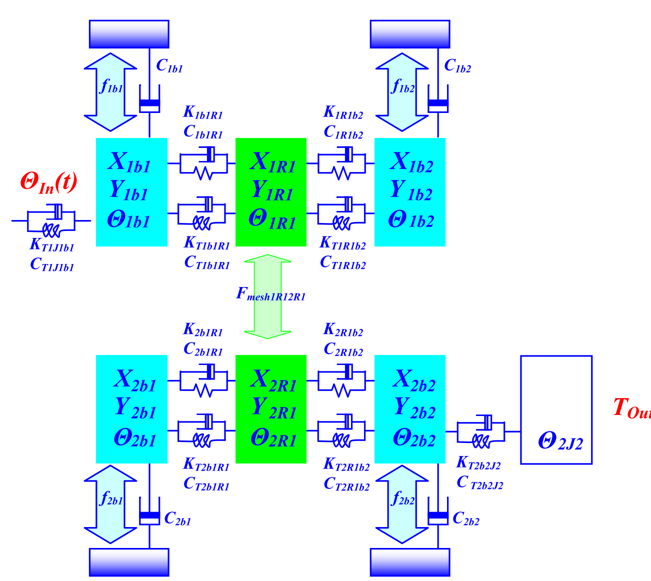

Fig. 1. Block diagram.