ARTICLE IN PRESS

Applied Acoustics xxx (2013) xxx–xxx

Contents lists available at ScienceDirect

Applied Acoustics

journal homepage: www.elsevier.com/locate/apacoust

Dynamic optimization design of a cylindrical helical spring

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ARTICLE INFO

Article history:
Available online xxxx

Keywords:
Helical springs
Hybrid mixed formulation
Natural frequencies
Dynamic response optimization

ABSTRACT

In this paper, a numerical method to model the dynamic behavior of an isotropic helical spring is coupled with optimization algorithms to construct a dynamic optimization method based not only on mechanical and geometrical objective functions and constraints; but also on dynamic ones. In the proposed dynamic optimization problem, four geometric parameters are chosen as design variables (wire diameter, middle helix diameter, active coils numbers and spring pitch). In addition of mechanical and geometrical constraints, dynamic ones related to natural frequencies of the helical spring are added. Two objective functions are chosen to be optimized: the spring mass and its natural frequencies. This method is then applied to the case of circular cross section helical spring, and then optimization results are presented and discussed.

1. Introduction

The optimization of the mechanical behavior of curved beams is an efficient tool in the design of mechanical components belonging to this kind of structures. One of these components, the helical spring which is one of the components used in many mechanical systems to provide a variety of functions (maintenance, suspension, energy accumulation, shock absorption, etc.). When a designer needs to incorporate a spring in the system, certain criteria must be known to help him to choose the appropriate spring. These criteria are more or less numerous and accurate according to the progress in the design process. For this, a new synthesis tools have recently appeared in the spring design. They used numerical optimization methods in order to give the user the opportunity to express freely their needs and to propose a spring respecting both its own criteria and all constraints. Therefore having a study of this component allows better design to meet the mechanical requirements needed in the industry.

Spring optimization techniques in the literature were made to minimize the mass, volume, stress distribution, movements according to certain geometric and mechanical parameters such as the space on which the spring works and the number of active coils. Kulkarni and Balasubrahmanyam [1] presented simplified graphs for the optimization of the helical spring by minimizing the mass, the free length and the volume. Yokota et al. [2] developed a technique for optimization a helical spring to minimize the mass taking into account the shear stress, the number of active coils, the spring wire diameter and the middle coil diameter using a genetic algorithm. Also Deb and Goyal [3] and Kannan and Kramer [4] compared their results based on the genetic algorithms and the augmented Lagrange method with results obtained by Sandregen [5] using a branch and bound approaches. Imaizumi et al. [6] and Hernandez [7] optimize the wire shape used for spring manufacture. Xiao et al. [8] introduced a new method for optimization of the helical spring based on the Particle Swarm Optimization algorithm and the minimum mass of helical spring as the objective function, with the geometric parameters as design variables and the shear stress, the maximum axial deflection, the critical frequency, the bucking, the fatigue strength, the condition of coils not touch, space and dimension are taken as constraint conditions.

Although the literature is rich of dynamic studies of helical spring, all these studies have not taken into account the dynamic parameters (natural frequencies, dynamic response ...) as constraints or as an objective function when they tried to optimize the mechanical behavior of the helical spring. In the first dynamic studies of helical springs, the dynamic response has been addressed by limiting the analysis to short displacements around the equilibrium position. When the spring becomes the subject of large impact load oscillations, its behavior becomes nonlinear. The equations of motion describing this behavior were derived by Philips and Costello [9]. Stokes [10] conducted analytical and experimental studies to investigate the spring radial displacement due to longitudinal impact. Mottershead [11] developed a finite element for solving differential motion equations. Yilidirim [12] studied the helical spring with circular and square sections by developing the stiffness matrix from the linear relationship between effort and strain, taking into account the effect of transverse shear. The resolution of the modal equation was made by the

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0003-682X/$ - see front matter © 2013 Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.apacoust.2013.08.001

method of subspace iteration to determine the natural frequencies of the spring. The influence of changes of some parameters (angle of the helix, middle coil diameter...) was studied. Forrester [13] analyzed the static and the dynamic behavior of the spring by a finite element and analytical methods to determine the stiffness and natural frequencies of the structure taking into account the curvature of the spring, the effects of shear and the geometric effects of spring section. These methods are based on solving differential equations with boundary conditions. In the first analysis, the spring was modeled by an assembly of beam elements. In the second analysis, the determination of the three-dimensional stiffness matrix of a helical spring was made. Taktak et al. [14] developed a two node finite element with six degrees of freedom per node, able to model the behavior of a three dimensional isotropic helical beam. Transverse shear and torsion effects and all geometric parameters are taking into account to study the spring dynamic response for harmonic excitations.

In this paper, an optimization method of the helical spring which incorporates the dynamic parameters as constraints and objective function to optimize the mechanical behavior of the helical spring in terms of its geometrical parameters (cross section, curvature, pitch ...) is presented. It is based on the numerical formulation presented in a previous work of Taktak et al. [14] to compute the dynamic response of the helical spring.

The outline is as follow: in the second section, the method used for calculating the dynamic response of the helical spring based on the modal superposition method is presented. In the third section, the proposed dynamic optimization method of the helical spring is presented by developing the objective functions and the design variables of the problem as well as the constraints conditions. This leads to establish a complex mathematic dynamic optimal design model of the helical spring. Finally, the problem is solved using an optimization algorithm, which is incorporated in MATLAB code. The numerical results were presented and discussed.

2. Dynamic analysis of the helical spring

The equation of motion of the helical spring is written as [14]:

$$[M]\{\ddot{U}\} + [C]\{\dot{U}\} + [K]\{U\} = \{F\} \tag{1}$$

$[M]$ and $[K]$ are respectively the helical spring mass and stiffness matrices [14–16]. $[C]$ is the total damping matrix. $\{U\}$ is the global nodal displacements vector and $\{F\}$ is the vector of external forces.

The description of the movement of this system with n degrees of freedom can be made by its spatial coordinates or by its modal coordinates. The movement's equation of the structure without a second member admits a complete linear orthogonal real modes basis of the non-damped system. These eigenmodes are characterized by the eigenpulsations ω_i and also by their eigenvectors $\{V_i\}$. The modal matrix is defined by [17]:

$$[\Phi] = [\{V_1\}, \{V_2\}, \ldots, \{V_{12}\}] \tag{2}$$

The projection of the motion's equation on the modal basis leads to build a system of n decoupled equations. The equation of the motion according to the generalized parameters is written as:

$$[M_m]\{\ddot{a}(t)\} + [C_m]\{\dot{a}(t)\} + [K_m]\{a(t)\} = \{F_m\} \tag{3}$$

where $a(t)$ is the generalized displacements vector defined as:

$$\{U(t)\} = [\Phi]\{a(t)\} \tag{4}$$

$$[M_m] = [\Phi]^T[M][\Phi] = diag(m_i) \tag{5}$$

is the generalized mass matrix.

$$[K_m] = [\Phi]^T[K][\Phi] = diag(m_i\omega_i^2) \tag{6}$$

is the generalized stiffness matrix.

$$[C_m] = diag(2m_i\omega_i\xi_i) \tag{7}$$

is the generalized damping matrix. ξ_i is the reduced modal damping coefficients [18]:

$$\{F_m\} = [\Phi]^T\{F\} = \begin{Bmatrix} f_1 \\ \vdots \\ f_2 \end{Bmatrix} \tag{8}$$

$\{F_m\}$ is generalized forces vector.

From Eq. (3) a system of the motion equations of n decoupled oscillators is obtained as follow:

$$m_i\ddot{a}_i + 2m_i\xi_i\omega_i\dot{a}_i + m_i\omega_i^2 a_i = f_i \qquad i = 1, \ldots, n \tag{9}$$

The modal frequency response of the variables $a_i(t)$ is simply the solution of n equations of motion transformed by Fourier is written:

$$m_i\omega_i^2 a_i(\omega) - m_i\omega^2 a_i(\omega) + 2jm_i\xi_i\omega_i\omega a_i(\omega) = f_i(\omega) \qquad i = 1, \ldots, n \tag{10}$$

The solution is:

$$a_i(\omega) = \frac{\frac{f_i(\omega)}{k_i}}{\left(1 - \left(\frac{\omega}{\omega_i}\right)^2\right) + 2j\xi_i\frac{\omega}{\omega_i}}; \quad i = 1, 2, \ldots, n \tag{11}$$

where

$$k_i = m_i\omega_i^2 \tag{12}$$

The frequency response of the system is the product of modal variables by modes:

$$\{U(\omega)\} = \sum_{i=1}^{n} a_i(\omega)\{U_i\} = \sum_{i=1}^{n} \frac{\frac{f_i(\omega)}{k_i}}{\left(1 - \left(\frac{\omega}{\omega_i}\right)^2\right) + 2j\xi_i\frac{\omega}{\omega_i}}\{U_i\} \tag{13}$$

3. Development of the dynamic optimization method of the helical spring

3.1. Design variables

In our study a circular cross section spring is studied. Four geometrical proprieties of the spring are chosen as design variables, these parameters are the wire diameter d, the middle helix diameter D, active coil number n_a and the helix pitch P. These parameters are presented in the design parameters vector:

$$\{X\} = \langle x_1 \quad x_2 \quad x_3 \quad x_4 \rangle^T = \langle d \quad D \quad n_a \quad P \rangle^T \tag{14}$$

The other parameters are supposed fixes.

3.2. Constraints conditions

In what follows, it is assumed that the following hypotheses are true:

- The wire section is and remains circular.
- The helix is slightly inclined ($\alpha < 7°$).
- The ends of the springs are ground and strengthened to make a perpendicular plans to the spring axis to support it without friction.

3.2.1. Condition of shear stress

To avoid the structure damage, when the spring is loaded with an axial force $\vec{F}$, the maximum shear stress τ_{max} should be less than the allowable shear resistance R_{pg} [8]:

Please cite this article in press as: Taktak M et al. Dynamic optimization design of a cylindrical helical spring. Appl Acoust (2013), http://dx.doi.org/10.1016/j.apacoust.2013.08.001