Applied Acoustics 76 (2014) 138–149

Contents lists available at ScienceDirect

Applied Acoustics

journal homepage: www.elsevier.com/locate/apacoust

Time domain finite volume method for three-dimensional structural–acoustic coupling analysis

Ling-kuan Xuan, Guo-yong Jin, Jing-feng Gong, Wen-ping Zhang, Ping-jian Ming *

College of Power and Energy Engineering, Harbin Engineering University, Harbin 150001, PR China

ARTICLE INFO

Article history:
Received 25 March 2013
Received in revised form 22 July 2013
Accepted 25 July 2013

Keywords:
Structural–acoustic coupling
Finite volume method
Three-dimensional
Time domain
Natural characteristics

ABSTRACT

This work extends the application of finite volume method (FVM) to structural–acoustic problems. A three-dimensional time domain FVM (TDFVM) is proposed to predict the transient response and natural characteristics of structural–acoustic coupling systems. Acoustic wave equation in heterogeneous medium and structural dynamic equation are solved in fluid and solid sub-domains respectively. The structural–acoustic coupling is implemented according to normal components of particle acceleration continuity condition and normal traction equilibrium condition at the interface. The computational domain is discretized with four-node tetrahedral grid which is generated easily and has strong adaptability to complicated geometries. Numerical experiments are carried out to examine the accuracy of the method in both time domain and frequency domain. The results show good agreement with analytical solutions and numerical results. For structural–acoustic problem, TDFVM has the capability to consider the heterogeneity of both fluid and solid.

1. Introduction

In noise control field, the structural–acoustic coupling problem is an important research topic which occurs in many engineering applications. Many researchers have investigated this problem and several analytical solutions have been provided [1–3] for regular coupling systems. However, it is impossible to obtain analytical solutions for the coupled systems in most practical problems. Therefore, much effort has been devoted to the development of numerical method. Toyoda and Takahashi [4] employed finite difference method (FDM) to predict the architectural structure-borne sound. However, the poor adaptability of FDM in irregular areas often restricts its application in complicated engineering problems. On the other hand, FEM owns well geometrical flexibility which makes it widely applied in structural–acoustic field [5–10].

Structural–acoustic coupling problem can be regarded as a particular fluid–structural–acoustic coupling case without flow. For such a multi-physics problem, it is necessary to adopt a unified numerical method [11]. Because transferring data from one solution procedure to another may lead to significant problems in computational convergence, which may even lead to computational divergence [12].

In fact, solid mechanics and fluid mechanics are governed by the same equation, but differ in constitutive relations [13]. The governing equation of acoustic wave is also derived from fluid basic equations. FEM has been applied to solve fluid–structure interaction problems [14–16]. But the low speed of FEM solution constrains its application in computational fluid dynamics (CFD) problems and multi-physics problems [17].

Finite volume method (FVM) has been extended to structural problems in recent years. Demirdžić et al. [13,18] first employed FVM for stress analysis. Wheel [19] applied FVM to analyze stress concentration problem and even showed FVM achieves better accuracy than FEM for a NAFEMS (National Agency for FEMs and Standards) steel elliptic membrane benchmark. Wheel [20] proposed a finite volume formulation for determining small deformations in incompressible materials under plane strain conditions which is still difficult for FEM. Slone et al. [12] presented a procedure to evaluate the dynamic structural response of elastic solid domain. Fallah et al. [21] developed the cell vertex and cell centered forms of FVM for plate bending analysis. The literatures mentioned above demonstrate the potential capability of FVM in solid problems. And considering the wide applications of FVM in CFD, FVM can be regarded as a unified alternative approach for structural–acoustic coupling problems which is rarely attempted.

This paper aims to extend the application of FVM to acoustic and structural–acoustic problems. This investigation employs a FVM approach developed from control volume finite element method (CVFEM). CVFEM was first proposed for convection diffusion problems by Baliga et al. [22]. It not only follows conservation laws like FVM but also owns the adaptability for irregular areas like FEM. Therefore, as soon as it was put forward the emphasis is on its

* Corresponding author. Address: No. 145 Nantong Street, Nangang District, Harbin City, Heilongjiang Province 15001, PR China. Tel.: +86 451 82569458.
E-mail address: pingjianming@hrbeu.edu.cn (P.-j. Ming).

0003-682X/$ - see front matter
http://dx.doi.org/10.1016/j.apacoust.2013.07.024

application and development. CVFEM is an effective numerical approach in FVM family to deal with the unstructured domain problems. It should be noted that there is no need to calculate and store the element stiffness matrix, so the computational cost and memory requirements are much less than FEM [23].

Structural–acoustic coupling problems can be solved in frequency domain (FD) and time domain (TD). The FD modeling algorithms are applied widely while the TD ones are less adopted compared with the former. It may be due to the less computational time cost by FD modeling. However, FD modeling consumes more computer memory for the storage of complex matrix. Hence, FD modeling algorithms may not be appropriate for large and complicated problems. On the other hand, TD modeling is more close to the real physical process from the standpoint of dynamics, since structural vibration or acoustic radiation is a kind of energy propagation in media. The entire information in the involved frequency band can be obtained within one simulation by TD modeling.

In this paper, time domain finite volume method (TDFVM) is developed based on unstructured staggered-grids to solve structural–structural, acoustic–acoustic and structural–acoustic coupling problems. The accuracy is validated by comparing the predicted results with analytical solutions and numerical results. Then TDFVM is applied to analyze the transient response and natural characteristics of a three-dimensional (3D) enclosed cavity, namely a structural–acoustic coupling system. The effects of the variation of water depth on natural characteristics of the system are discussed. A different material cavity filled with air and water is also analyzed.

2. Mathematic models

Let Ω_S and Ω_F be the structural and acoustic sub-domains, respectively, as shown in Fig. 1. The structural–acoustic interface is denoted by Γ_{FSI}. The boundaries of Ω_S contain clamped boundary Γ_C, force boundary Γ_f and free boundary Γ_N. $\boldsymbol{n_s} = (n_x, n_y, n_z)$ is the normal vector of the structural sub-domain. The acoustic sub-domain consists of air and water. The air–water interface is denoted by Γ_{FI}.

According to the Newton's second law, the equilibrium equation in a control volume of the structural sub-domain is

$$\int_{\Omega_S} \rho_S \ddot{\mathbf{u}} d\Omega = \int_{\Omega_S} \nabla \cdot \boldsymbol{\sigma} d\Omega + \int_{\Omega_S} \boldsymbol{F} d\Omega \tag{1}$$

where ρ_S is the material density, $\boldsymbol{u} = (u_x, u_y, u_z)$ is the acceleration vector, $\boldsymbol{\sigma}$ is the Cauchy stress tensor, and $\boldsymbol{F}$ is body force which is omitted in the following equations.

The constitutive equations for isotropic linear elastic body are as follows:

$$\begin{aligned}
\sigma_{xx} &= 2G\frac{\partial u_x}{\partial x} + \lambda\left(\frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} + \frac{\partial u_z}{\partial z}\right), \quad \sigma_{xy} = G\gamma_{xy} \\
\sigma_{yy} &= 2G\frac{\partial u_y}{\partial y} + \lambda\left(\frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} + \frac{\partial u_z}{\partial z}\right), \quad \sigma_{zy} = G\gamma_{zy} \\
\sigma_{zz} &= 2G\frac{\partial u_z}{\partial z} + \lambda\left(\frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} + \frac{\partial u_z}{\partial z}\right), \quad \sigma_{xz} = G\gamma_{xz}
\end{aligned} \tag{2}$$

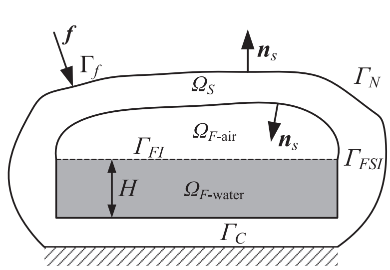

Fig. 1. Structural–acoustic coupled system.

where λ and G are the Lamé constants as $\lambda = E\mu/[(1+\mu)(1-2\mu)]$, $G = E/[2(1+\mu)]$. E is the Young's modulus and μ is the Poisson's ratio.

The integral form of acoustic wave equation in heterogeneous media can be expressed by [24]

$$\int_{\Omega_F} \frac{1}{\rho_F c^2} \frac{\partial^2 p}{\partial t^2} d\Omega = \int_{\Omega_F} \nabla \cdot \left(\frac{1}{\rho_F} \nabla p\right) d\Omega \tag{3}$$

where p is the acoustic pressure, ρ_F is the media density and c is the acoustic wave speed.

The interaction between the structural and acoustic sub-domains is carried out by

$$-p\boldsymbol{n} = \boldsymbol{\sigma} \cdot \boldsymbol{n} \tag{4}$$

$$-\nabla p / \rho_F = \ddot{\mathbf{u}} \tag{5}$$

According to Eq. (4), when the acoustic wave meets the structural–acoustic interface Γ_{FSI}, the acoustic pressure p acts on the interface just like a force. At the same time, when stress wave reaches the interface, the structural acceleration causes the oscillation of the particles in the acoustic medium, and the expression of Eq. (5) is similar to the momentum equation of the acoustic wave.

In structural sub-domain, two types of boundary conditions are considered as

$$(\boldsymbol{\sigma} \cdot \boldsymbol{n})_{\Gamma_N} = 0 \tag{6}$$

$$(\boldsymbol{u})_{\Gamma_C} = 0 \tag{7}$$

where $\boldsymbol{u} = (u_x, u_y, u_z)$ denotes the displacement vector.

In acoustic sub-domain, there are two kinds of total-reflecting boundary conditions [25]. When acoustic wave incidences from air to water, it can be treated approximately as $v = 0$ at the interface where v is the normal velocity, namely $\partial p/\partial n = 0$. On the other hand, when acoustic wave incidences from water to air, it can be treated approximately as $p = 0$ at the interface.

3. Numerical methods

In this section, numerical discretization for structural and acoustic sub-domains by TDFVM is discussed in detail. Then the formulations and algorithms for the structural–acoustic coupling are presented.

3.1. Structural sub-domain

The 3D computational domain is divided by tetrahedrons and an arbitrary tetrahedron G_{1234} is shown in Fig. 2. Acceleration, velocity and displacement are calculated on the vertexes 1, 2, 3, 4, and the stress in G_{1234} is supposed to be uniform. Material properties are defined at the grid center with uniform distribution in the grid. Therefore, the left hand side of Eq. (1) can be read in the following form based on the assumption of a uniform distribution of the acceleration in the control volume around vertex 1

$$\int_V \rho_S \ddot{\mathbf{u}} dV = \rho_S \ddot{\mathbf{u}} V \tag{8}$$

where $V = \sum_{i=1}^{n} (V_i/4)$ is the volume of the control volume, and V_i is the volume of the ith tetrahedron grid around vertex 1.

Invoking the Gauss theorem, the right hand side of Eq. (1) becomes

$$\int_V \nabla \cdot \boldsymbol{\sigma} dV = \oint_s \boldsymbol{\sigma} \cdot \boldsymbol{n} dA = \sum_{i=1}^{N} (\boldsymbol{\sigma} \cdot \boldsymbol{A}_1)_i \tag{9}$$