



Vibration of strings with nonlinear supports

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ABSTRACT

The dynamic string motion, which displacement is unilaterally constrained by the rigid termination condition of an arbitrary geometry has been simulated and analyzed. The treble strings of a grand piano usually terminate at a capo bar, which is situated above the strings. The apex of a V-shaped section of the capo bar defines the end of the speaking length of the strings. A numerical calculation based on the traveling wave solution is proposed for modeling the nonlinearity inducing interactions between the vibrating string and the contact condition at the point of string termination. It was shown that the lossless string vibrates in two distinct vibration regimes. In the beginning the string starts to interact in a nonlinear fashion with the rigid terminator, and the resulting string motion is aperiodic. Consequently, the spectrum of the string motion depends on the amplitude of string vibrations, and its spectral structure changes continuously with the passage of time. The duration of that vibration regime depends on the geometry of the terminator. After some time of aperiodic vibration, the string vibrations settle in a periodic regime where the resulting spectrum remains constant.

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1. Introduction

Investigation of the boundary condition of vibrating string is a very important problem in musical acoustics. It is well known that the fundamental frequency of the string is strictly determined by the type of the string termination. Usually the changing of the tone caused by the curvature of the string support is negligible, but there is a family of Japanese plucked stringed instruments (*biwa* and *shamisen*), which sounding is strictly determined by the string termination [1,2]. These lutes are equipped with a mechanism called “*sawari*” (touch). The *sawari* is a contact surface of very limited size, located at the nut-side end of the string, to which the string touches repeatedly, producing a unique timbre of the instrumental tone called the *sawari* tone.

There are other stringed instruments of Indian origin with a similar bridge design, such as sitar, veena and tambura. The interaction of the string with a curved string support creates a peculiar buzzing sound, which is markedly different from that of known European plucked string instruments such as guitar and lute. The geometry of the string terminations for the sitar, veena, and tambura was considered by Raman [3]. Raman concluded that possible explanation of the phenomena of the “missing modes” is the complex interaction of the string with the bridge [4].

Much effort has been devoted to modeling the dynamics of a vibrating string with a distributed unilateral constraint during the past decades. This problem was considered by Schatzman [5],

Burridge et al. [6], and Cabannes [7] who used the method of characteristics, and assumed that the string does not lose energy when it hits the obstacle. Krishnaswamy and Smith [8], Han and Grosenbaugh [9], and Taguti [10] used a finite difference method to study the string interaction with the curved bridge. Vyasrayani et al. [11] described the movement of the sitar string with a set of partial differential equations. Rank and Kubin [12], Evangelista and Eckerholm [13], and Siddiq [14] used a waveguide modeling approach to study the plucked string vibration with nonlinear limitation effects.

The present paper describes a physics-based model for simulation of vibrations of piano string, which at one end has the ideal rigid support, and its other end is terminated at a capo bar. The types of the string support in the piano are different for the bass and treble notes. All the far ends of the piano strings are terminated at the bass and treble bridges, which are rather complicated resonant systems. The nearest ends of the bass and long treble strings begin at the agraffe that can be considered as an absolutely rigid clamp termination. However the treble strings of grand pianos start at the capo bar – the rigid edge of the cast iron frame [15]. These strings are bent around the capo bar, and their vibration tone depends on the curvature of the capo bar V-shaped section. The same type of the string support can be seen also on the guitar and some other musical string instruments.

The aim of this paper is to show the influence of the contact nonlinearity on the spectral structure of the piano string vibration. A part of this analysis was presented in [16]. The study is divided into two stages. Firstly, the mathematical modeling of the hammer–string interaction enables prediction of the piano string

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motion [17,18]. Secondly, this knowledge is used for appropriate simulation of interaction of the vibrating string with a capo bar.

The numerical simulation of the hammer–string interaction is based on the physical models of a piano hammer described in [19–21]. These models are based on the assumption that the woolen hammer felt is a microstructural material possessing history-dependent properties. The elastic and hereditary parameters of piano hammers were obtained experimentally using a special piano hammer testing device that was developed and built in the Institute of Cybernetics at Tallinn University of Technology [21].

In this paper a number of simplifying assumptions regarding the string and string support are introduced. Thus, the piano string is assumed to be an ideal flexible string, the coupling of strings at the end supports is neglected, and the bridge motion is ignored. We also assume that the right string termination (bridge) is the ideal rigid support. The left string termination (capo bar, *sawari*) is considered here as a rigid but not an ideal support, because we take into account the curvature of its V-shaped section. Nevertheless, we hope that the application of the proposed model will clarify the physics of vibration of the string with nonlinear support.

2. First stage. String with ideal rigid support

2.1. Piano string model

It is assumed that the piano string is an ideal (flexible) string. The transverse displacement $y(x, t)$ of such a string obeys wave equation

$$\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}. \quad (1)$$

Like in [17,18], we have the system of equations describing the hammer–string interaction

$$\frac{dz}{dt} = -\frac{2T}{cm}g(t) + V, \quad (2)$$

$$\frac{dg}{dt} = \frac{c}{2T}F(t), \quad (3)$$

where function $g(t)$ is the form of outgoing wave created by the hammer strike at the contact point $x = l$, c is the speed of a nondispersive wave traveling along the string; $F(t)$ is the acting force, T is the string tension; m , $z(t)$, and V are the hammer mass, the hammer displacement, and the hammer velocity, respectively. The hammer felt compression is defined by $u(t) = z(t) - y(l, t)$. Function $y(l, t)$ describes the string transverse displacement at the contact point $x = l$, and is given by [18]

$$y(l, t) = g(t) + 2 \sum_{i=1}^{\infty} g\left(t - \frac{2il}{c}\right) - \sum_{i=0}^{\infty} g\left[t - \frac{2(i+a)L}{c}\right] - \sum_{i=0}^{\infty} g\left[t - \frac{2(i+b)L}{c}\right]. \quad (4)$$

Here we suppose that the string of length L extends from $x = 0$ on the left to $x = L$. Parameter $a = l/L$ is the fractional length of the string to the striking point, and $b = 1 - a$. Parameter a determines the actual distance l of the striking point from the nearest string end. The initial conditions at the moment when the hammer first contacts the string, are taken as $g(0) = z(0) = 0$, and $dz(0)/dt = V$.

The physical interpretation of Eq. (4) is simple enough. It describes the deflection of the string at the contact point that is determined by the traveling waves moving in both directions along the string and reflecting back from the string supports. Here the index of summation i simply denotes the number of reflections.

2.2. Piano hammer model

The experimental testing of piano hammers demonstrates that all hammers have a hysteretic type of force–compression characteristics. A main feature of hammers is that the slope of the force–compression characteristics is strongly dependent on the rate of loading. It was shown that nonlinear hysteretic models can describe the dynamic behavior of the hammer felt [19–21]. These models are based on assumption that the hammer felt made of wool is a microstructural material possessing history-dependent properties. Such a physical substance is called either a hereditary material or a material with memory.

According to a three-parameter hereditary model of the hammer presented in [20], the nonlinear force $F(t)$ exerted by the hammer is related to the felt compression $u(t)$ by the following expression

$$F(u(t)) = Q_0 \left[u^p + \alpha \frac{d(u^p)}{dt} \right]. \quad (5)$$

Here the parameter Q_0 is the static hammer stiffness; p is the compliance nonlinearity exponent, and α is the retarded time parameter.

The continuous variations in hammer parameters across the compass of the piano were obtained experimentally by measuring a whole hammer set of recently produced unvoiced *Abel* hammers. The result of those experiments is presented in [20,21]. A best match to the whole set of hammers $1 \leq n \leq 88$ was approximated using

$$Q_0 = 183 \exp(0.045n), \quad (6)$$

$$p = 3.7 + 0.015n, \quad (7)$$

$$\alpha = 259.5 + 0.58n + 6.6 \cdot 10^{-2}n^2 - 1.25 \cdot 10^{-3}n^3 + 1.172 \cdot 10^{-5}n^4. \quad (8)$$

Here the unit for parameter α is ms, and the unit for Q_0 is N/mm^p.

The hammer masses of this set were approximated by

$$m = 11.074 - 0.074n + 10^{-4}n^2, \quad 1 \leq n \leq 88. \quad (9)$$

The mass of hammer 1 (A_0) is 11.0 g, and the mass of hammer 88 (C_8) 5.3 g.

2.3. Numerical simulation for tone A_7

The hammer–string interaction is simulated by solving the system of Eqs. (2, 3) for various initial hammer velocities. We chose for calculations the note number $n = 85$ (tone A_7 , frequency $f = 3520$ Hz). The string parameters are the following: the string length $L = 61$ mm; the actual distance of the striking point from nearest string end $l = 2.6$ mm; the linear mass density of the string $\mu = 4.2$ g/m; the string mass $M = 0.26$ g; the string tension $T = 774.6$ N.

For tone A_7 in grand pianos there are three strings per note, the acting mass of a hammer defined by relation (9) for $n = 85$ is chosen equal to 1/3 of the total hammer mass, and thus $m = 1.8$ g. For the hammer 85 we use the following additional parameters: static stiffness $Q_0 = 8387.4$ N/mm^p; nonlinearity exponent $p = 4.975$; hereditary parameter $\alpha = 0.5312$ ms.

As a result of simulation of a hammer–string interaction we can find the history of the acting force $F(t)$ and the time dependence of the outgoing wave $g(t)$ created by the hammer strike, which are shown in Fig. 1.

At the moment $t = t_0$, which is defined as the duration of contact, the hammer has lost the contact with the string. After this moment the acting force $F(t) = 0$ for any time $t > t_0$. Therefore, according to Eq. (3), the outgoing wave $g(t) = \text{const}$ for the moments $t > t_0$.

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