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# Free convective micropolar fluid flow and heat transfer over a shrinking sheet with heat source

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ABSTRACT

The free convective micropolar fluid over a shrinking sheet in presence of heat source/sink has been studied in this paper. The method of solution involves similarity transformation. The coupled non-linear partial differential equations representing momentum and concentration and non homogeneous heat equation are reduced into set of non-linear ordinary differential equations. The transformed equations are solved by applying Runge-Kutta method followed by shooting technique. The effect of pertinent parameters characterizing the flow has been presented through the graphs and then discussed. It is found that heat source has significant effect on velocity profile and causes a decrease in the boundary layer. Angular velocity profile increases with the increasing value of material parameter. Present study showed an excellent agreement with published literature.

## 1. Introduction

The boundary layer flow over a shrinking surface is encountered in several technological processes. Such situations occur in polymer processing, manufacturing of glass sheets, paper production, in textile industries and many others. The boundary layer flows of a non-Newtonian fluid over a stretching sheet are always important from engineering point of view. It has wide applications in extrusion process, glass fibre and paper production, wire drawing, food processing and movement of biological fluids. The heat transfer analysis of a boundary layer flow with radiation is further important in electrical power generation, astrological flows, solar power technology, space-vehicle re-entry and other industrial areas. The theory of micropolar fluids has drawn much attention during recent years due to its importance in many technological applications such as cooling of nuclear reactors during emergency shutdown, cooling electronic devices, enhancing oil recovery etc. Eringen [1] developed the theory of micropolar fluid which cannot be explained by classical Navier-stoke equations due to micro-inertia and spin or microrotational effects. Ahmadi [2] found out the self-similar solution of incompressible

micropolar boundary layer flow over a semi-infinite plate. Micropolar boundary layer flow at a stagnation on a moving wall was investigated by Gorla [3].

The problem of fluid past a stretching sheet has received wide range of attention because of its technological applications in the field of metallurgy and chemical engineering. Crane [4] was first to give an analytical solution for laminar boundary layer flow past a stretching sheet. Shankara et al. [5] studied the micropolar flow past a stretching sheet. Magnetohydrodynamic flow of a power-law

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fluid over a stretching sheet was discussed by Cortell [6]. Wang [7] presented a study on liquid film on an unsteady stretching sheet. Kumaran et al. [8] solved the problem of transition of MHD boundary layer flow past a stretching sheet using finite difference method of Crank-Nicholson type. MHD non-Darcy mixed convective diffusion of species over a stretching sheet embedded in a porous medium with non-uniform heat source/sink, variable viscosity and soret effect was studied by Pal and Mondal [9]. They had used the fifth order Runge-Kutta-Felhberg method for this work. Radiation and magnetic field effects on the unsteady mixed convection flow of a second grade fluid over a vertical stretching sheet were reported by Hayat and Qasim [10]. Mishra et al. [11] investigated flow of heat and mass transfer on MHD free convection in a micropolar fluid with heat source. Recently, Mohanty et al.[12] studied on heat and mass transfer effect of micropolar fluid over a stretching sheet.

The flow induced by a shrinking sheet shows physical phenomena quite distinct from the forward stretching flow. The work on shrinking sheet is generalized to include a stagnation flow which is a backward flow as discussed by Goldstein. Miklavcˇicˇand Wang [13] studied viscous flow due to a shrinking sheet. In their paper exact solution of Navier-Stoke equations was presented and it was shown that mass suction is required to maintain the flow over a shrinking sheet. Hayat et al. [14] reported the analytic solution for MHD rotating flow of a second grade fluid over a shrinking surface. In another work they [15] investigated MHD flow and mass transfer of an upper-convected Maxwell fluid past a porous shrinking sheet with chemical reaction species. Fang and Zhang [16] reported closed-form exact solution of MHD viscous flow over a shrinking sheet. Non-perturbative solution for MHD viscous flow due to a shrinking sheet was analysed by Noor et al. [17]. Ishak [18] et al. presented a study on stagnation-point flow over a shrinking sheet in a micropolar fluid. They [19] also extended their work to non-Newtonian power-law fluid flow past a shrinking sheet with suction. Several researchers [20–25] worked on non-Newtonian fluid flow past a shrinking sheet with different models. They approached this problem both analytically and numerically. Afridi and Qasim [26,27] studied entropy generation in two different heat transfer problems. Qasim and Afridi [28] examined effects of energy dissipation and variable thermal conductivity on entropy generation rate in mixed convection flow. Recently, Dash et al. [29] analysed numerical approach to boundary layer stagnation-point flow past a stretching/shrinking sheet. Bhattacharyya et al. [30] found out the dual solutions for the effects of thermal radiation on micropolar fluid flow and heat transfer over a porous shrinking sheet.

In our present study, the steady flow of micropolar fluid with heat transfer and thermal radiation over a porous shrinking sheet is considered in the presence of heat source. The solution of the present problem is obtained by Runge-Kutta method followed by shooting technique.

## 2. Formulation of the problem

A two-dimensional steady flow of micropolar fluid with heat transfer and thermal radiation over a porous shrinking sheet is considered. The shrinking velocity of the sheet is $U_w = -cx$ where $c$ is the shrinking constant such that $c > 0$. The equations of motion for the micropolar fluid and heat transfer under the boundary layer approximation are [11,12]:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = (\upsilon + \frac{\kappa}{\rho})\frac{\partial^2 u}{\partial y^2} + \frac{\kappa}{\rho}\frac{\partial N}{\partial y} + g\beta(T - T_\infty) \tag{2}$$

$$u\frac{\partial N}{\partial x} + v\frac{\partial N}{\partial y} = \frac{\gamma}{\rho j}\frac{\partial^2 N}{\partial y^2} - \frac{\kappa}{\rho j}\left(2N + \frac{\partial u}{\partial y}\right) \tag{3}$$

$$\rho c_p\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = \kappa^*\frac{\partial^2 T}{\partial y^2} - \frac{\partial q_r}{\partial y} + Q^*(T - T_\infty) \tag{4}$$

Subject to the boundary conditions:

$$\left.\begin{array}{lll} u = U_w = -cx,\ v = v_w,\ N = -m\frac{\partial u}{\partial y},\ T = T_w, & & at\ y = 0 \\ u \to 0, \quad N \to 0, \quad T \to T_\infty, & & as\ y \to \infty \end{array}\right\} \tag{5}$$

The kinematic fluid viscosity,$\upsilon(=\mu/\rho)$ has a direction normal to the xy-plane, $T_w$ and $T_\infty$ both are assumed to be constant. Here, $v_w < 0$ for mass suction and $v_w > 0$ for mass injection. We note that m is a constant such that $0 \le m \le 1$. The case $m = 0$ indicates $N = 0$ at the surface. It represents flow of concentrated particle in which the microelements closed to the wall surface are unable to rotate. This case is also known as strong concentration of microelements. The case $m = 0.5$ indicates the vanishing of the anti-symmetric part of the stress tensor and denotes weak concentration of microelements. Whereas, the case $m = 1$ is used for the modelling of turbulent boundary layer flows. We assume that the spin gradient viscosity ($\gamma$):

$$\gamma = (\mu + \kappa/2)j = \mu(1 + \frac{K}{2})j \tag{6}$$

where $K = \frac{\kappa}{\mu}$ is the material parameter. This assumption is invoked to allow the field of equations to predict the correct behaviour in the limiting case when the microstructure effects become negligible and the total spin N reduces to the angular velocity.

Using Rosseland's approximation for radiation, we obtain $q_r = -(\frac{4\sigma}{3k_1})\frac{\partial T^4}{\partial y}$, where $\sigma$ is the Stefan-Boltzmann constant, $k_1$ is the