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Takagi-Sugeno fuzzy model identification for turbofan aero-engines with guaranteed stability

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Abstract This paper is concerned with identifying a Takagi-Sugeno (TS) fuzzy model for turbofan aero-engines working under the maximum power status (non-afterburning). To establish the fuzzy system, theoretical contributions are made as follows. First, by fixing antecedent parameters, the estimation of consequent parameters in state-space representations is formulated as minimizing a quadratic cost function. Second, to avoid obtaining unstable identified models, a new theorem is proposed to transform the prior-knowledge of stability into constraints. Then based on the aforementioned work, the identification problem is synthesized as a constrained quadratic optimization. By solving the constrained optimization, a TS fuzzy system is identified with guaranteed stability. Finally, the proposed method is applied to the turbofan aero-engine using simulation data generated from an aerothermodynamics component-level model. Results show the identified fuzzy model achieves a high fitting accuracy while stabilities of the overall fuzzy system and all its local models are also guaranteed.

1. Introduction

Turbofan aero-engine, designed to work safely and efficiently under a wide range of operating conditions, plays a pivotal role in the flight capabilities of modern aircrafts. To achieve a consistent transient response under a variety of working conditions, the gain scheduling technology is utilized extensively in engine control systems. The main idea of gain scheduling is to decompose the complex nonlinear aero-engine system into a set of linear systems by selecting break points. Then controllers are designed for each break point to cover all the operating conditions. As a consequence, a large number of break points have to be selected such that the system behavior between adjacent breakpoints is applicable for linear interpolations. Moreover, due to the lack of a systematic method, designs and validations of the controllers are still exhausting efforts. To facilitate the controller design, the Takagi-Sugeno (TS) fuzzy control is found to be a promising solution which motivates us to establish a TS fuzzy model for the aero-engine.

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Nomenclature

$\boldsymbol{z}$, $\boldsymbol{u}$, $\boldsymbol{x}$, $\boldsymbol{y}$	premise variables (flight conditions), inputs, states, outputs
N_{r}, N_l, N_k N_{data}	number of the rule, the sampled measurement, the sampling points, the flight condition
N_z, N_u, N_x, N_y	number of the elements in $\boldsymbol{z}$, $\boldsymbol{u}$, $\boldsymbol{x}$, $\boldsymbol{y}$
u_m, x_n, y_r	the mth, nth and rth element in $\boldsymbol{u}$, $\boldsymbol{x}$, $\boldsymbol{y}$ respectively
$(\boldsymbol{x}_j^{\mathrm{ss}}, \boldsymbol{y}_j^{\mathrm{ss}}, \boldsymbol{u}_j^{\mathrm{ss}})$	steady-state value under the jth flight condition
$\boldsymbol{G}_i$, $\boldsymbol{H}_i$, $\boldsymbol{C}_i$, $\boldsymbol{D}_i$	system matrices of the ith rule
$\mu_i(z)(i = 1, 2, \ldots, N_{\mathrm{r}})$	normalized firing strength of ith rule
$M_{i1}, M_{i2}, \ldots, M_{iN_z}$	N_z fuzzy sets of the ith rule
$\delta\boldsymbol{x}$, $\delta\boldsymbol{y}$, $\delta\boldsymbol{u}$	deviation from the steady-state point value
$\delta\boldsymbol{u}_{jl}$, $\delta\boldsymbol{x}_{jl}$, $\delta\boldsymbol{y}_{jl}$	the subscript jl denotes the lth measurement sampled under the jth flight condition
$\delta u_{m,jl}$, $\delta x_{n,jl}$, $\delta y_{r,jl}$	the mth, nth and rth element in $\delta\boldsymbol{u}_{jl}$, $\delta\boldsymbol{x}_{jl}$ and $\delta\boldsymbol{y}_{jl}$ respectively
$\boldsymbol{Y}_{jl}$, $\boldsymbol{V}_{jl}$, $\boldsymbol{U}_{jl}$	augmented matrix obtained by concentrating $\delta\boldsymbol{u}_{jl}$, $\delta\boldsymbol{x}_{jl}$ and $\delta\boldsymbol{y}_{jl}$.
$\boldsymbol{\Lambda}$	underdetermined matrix obtained by concentrating $\boldsymbol{G}_i$, $\boldsymbol{H}_i$, $\boldsymbol{C}_i$ and $\boldsymbol{D}_i$
†	Moore-Penrose pseudo-inverse
$\lVert\,\rVert_{\mathrm{F}}$	Frobenius norm
$\mathrm{vec}(\boldsymbol{A})$	yielding a vector by stacking the columns of $\boldsymbol{A}$
$\otimes$	Kronecker product
tr	trace of a matrix
$\mathrm{col}_{i=1}^{n}(\boldsymbol{G}_i)$, $\mathrm{row}_{i=1}^{n}(\boldsymbol{G}_i)$, $\mathrm{diag}_{i=1}^{n}(\boldsymbol{G}_i)$	concentrating matrices $\boldsymbol{G}_i(i = 1, 2, \ldots, n)$ along the vertical, horizontal and diagonal direction
*	an ellipsis for the symmetric terms in symmetric matrices
$J(\boldsymbol{\Lambda})$	function to be minimized
$\mathcal{R}_s$	the sth subspace
N_{s}	number of the subspaces
$\mathcal{A}(s)$	a set containing the index s and all the indexes of subspaces being adjacent to $\mathcal{R}_s$
$\mathcal{I}(s)$	a set containing all the indexes of system matrices firing in $\mathcal{R}_s$
λ_1, λ_2	eigenvalue

The TS fuzzy model proposed by Takagi and Sugeno[1] is described by the following IF-THEN rules which represent the local input-output relationship of a nonlinear system:

IF antecedent proposition THEN consequent proposition.

Then the overall system is achieved by fuzzy blending of these local models. Generally, a TS fuzzy model is established by linearizing the given nonlinear model around its operating points. However, as in many engineering applications, an explicit mathematical model of the aero-engine is very hard to obtain. Thus, the data-driven identification method is preferred in this paper.

When performing identification, antecedents and consequents of the rules are usually identified separately. Among them, efforts are made majorly on the latter by various methods, like fuzzy clustering[2,3] or evolving methods.[4–6] This is because with fixed antecedent parameters, identifying parameters of the consequent models is formulated as a linear least-square problem, which can be solved analytically and optimally.[7] However, in existing studies[8–11] and references therein, only Nonlinear AutoRegressive eXogenous (NARX) types are considered as the form of the consequents. To the best of our knowledge, the state-space representation which is favored in most control relevant studies has not been formulated so far.

Besides, due to reasons like non-persistent exciting signals, measurement noises or even inappropriate partitions of the antecedent space, the identified model may be unstable. However, only a little attention has been paid on the stability of the identified fuzzy systems. Among them, Abonyi et al.[12] transforms the prior-knowledge of the process stability into linear inequality constraints when identifying a TS fuzzy system of the NARX type. As for the identification of linear models, methods like augmenting the data[13] or the cost function[14] have been developed to obtain a stable representation at a cost of distorting the data or the cost function. By contrast, Lacy and Bernstein[15] guarantees the stability by introducing additional constraint. As a result, a specific weighting matrix should be taken therein. However, this is not a common case since the weighting matrix usually has a physical meaning and therefore cannot be designated casually. Inspired by Lacy and Bernstein's method, stability constraints of innovative forms are proposed in this paper for the TS fuzzy system, which includes the local linear system as a special case.

To sum up, the contribution of this paper is threefold. First, given fixed antecedent parameters, the problem of identifying a TS fuzzy system of state-space representation is formulated and has not been established before. Then based on this representation, a new theorem is proposed to ensure the global asymptotic stability of the identified TS fuzzy system and all its local models. Finally, the proposed method is utilized to identify a TS fuzzy system for the turbofan aero-engines, which facilitates the application of the fuzzy control.

This paper is organized as follows: the preliminary is presented in Section 2, and identifying the TS fuzzy system with guaranteed stability is formulated in Section 3. Then numerical example of a turbofan aero-engine is given in Section 4. Finally, conclusion is presented in Section 5.

2. Preliminary

The ith local discrete linear model of the TS fuzzy model can be described as follows:

$$\text{Rule } i: \text{ IF } z_1 \text{ is } M_{i1}, z_2 \text{ is } M_{i2}, \ldots, \text{ and } z_{N_z} \text{ is } M_{iN_z} \text{ THEN}$$
$$\begin{cases} \delta\boldsymbol{x}(k+1) = \boldsymbol{G}_i \cdot \delta\boldsymbol{x}(k) + \boldsymbol{H}_i \cdot \delta\boldsymbol{u}(k) \\ \delta\boldsymbol{y}(k) = \boldsymbol{C}_i \cdot \delta\boldsymbol{x}(k) + \boldsymbol{D}_i \cdot \delta\boldsymbol{u}(k) \end{cases} \quad (1)$$

where $i = 1, 2, \ldots, N_{\mathrm{r}}$ is the rule index, $\boldsymbol{z} = [z_1, z_2, \ldots, z_{N_z}] \in \boldsymbol{R}^{N_z}$ is the premise variables vector and $M_{i1}, M_{i2}, \ldots, M_{iN_z}$ are corresponding fuzzy sets. $\boldsymbol{x}(k) \in \boldsymbol{R}^{N_x}$ is the state vector. $\boldsymbol{u}(k) \in \boldsymbol{R}^{N_u}$ is the input vector. $\boldsymbol{y}(k) \in \boldsymbol{R}^{N_y}$ is