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Attitude control without angular velocity measurement for flexible satellites

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Abstract In this paper, by using quaternion models, the problem of attitude control is investigated for a class of flexible satellites. Two control laws are presented for the considered flexible satellite models to guarantee convergence of the closed-loop systems without using angular velocity measurement. One is in the form of a partial state feedback for the case where the modal variable is available, and the other is in the form of an observer-based partial state feedback for the case where the modal variable cannot be measured. Finally, an example is employed to illustrate the effectiveness of the proposed control laws.

1. Introduction

The attitude control of spacecrafts and satellites has important applications for some space missions such as pointing and formation flying. This topic has attracted much attention from a considerable number of researchers.[1–4] For attitude control, much investigation is based on the unit quaternion representation.[5–8] In Ref. 5, some attitude controllers with the structure of a Proportional-Derivative (PD) feedback plus feed-forward were designed for a rigid body. In Ref. 6, a sliding mode control law was designed and applied to spacecraft attitude tracking maneuvers when the inertia of a spacecraft was not exactly known. In Ref. 7, attitude control was considered for a rigid spacecraft with the control signal constrained by a common maximum magnitude in the presence of bounded unknown disturbances. A sliding mode controller was designed for such a type of spacecrafts to achieve global stability. The designed controller was in the form of a proportional feedback plus a smooth switch-like feedback with an auxiliary time-varying attitude gain function. High-order sliding mode controllers were designed in Ref. 8 for attitude control of a rigid spacecraft. A merit of the designed controller is that the phenomena of chattering can be eliminated. In Ref. 9, two Fault-Tolerant Control (FTC) schemes were derived for spacecraft attitude stabilization with external disturbances. In Ref. 10, the quaternion model of a rigid spacecraft was firstly transformed into

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the Lagrange-like, and then two robust sliding mode controllers were proposed to solve attitude tracking problems in the absence of both model uncertainties and external disturbances as well as in their presence.

In the aforementioned attitude control laws, the angular velocity of a rigid spacecraft was used in the construction of an attitude control law. However, in some circumstances, it is not easy to measure the angular velocity. Therefore, it is necessary to design an attitude control law without angular velocity measurement. Such a controller was designed in Ref. 11 by using a nonlinear filter of the quaternion to replace the angular velocity feedback. In Ref. 12, a velocity-free attitude stabilization scheme was proposed for a rigid spacecraft. In this control scheme, an angular velocity observer-like system was explicitly designed to construct the stabilizing feedback. In Ref. 13, two simple Saturated Proportional-Derivative (SPD) controllers were proposed for asymptotic stabilization of a rigid spacecraft with actuator constraints and without velocity measurement. In Ref. 14, a continuous angular velocity observer with fractional power functions was proposed to estimate the angular velocity via quaternion attitude information.

For flexible spacecraft, the effect of the motion of the elastic appendages must be taken into consideration, and thus the attitude control problem is more complicated. In Ref. 15, a dynamic controller was proposed for the attitude control of a flexible spacecraft under the assumption that the modal variables describing flexible elements were not available. In Ref. 16, an adaptive sliding mode control law with a hybrid sliding surface was proposed for a flexible spacecraft to minimize the effect of uncertainties and disturbances. In Ref. 17, an adaptive control law was proposed to solve the attitude tracking problem for flexible spacecrafts subject to a gravity-gradient disturbance under inertia matrix uncertainty. In Ref. 18, a nonlinear observer-based state feedback control law was designed to ensure the control objectives for attitude tracking. In Refs. 15–18, attitude control laws for flexible spacecraft were designed based on the unit quaternion representation. In Ref. 19, the three-axis attitude tracking control problem was investigated in presence of parameter uncertainties and disturbances based on the modified Rodrigues parameterization. An attitude control law was presented in the form of a nonlinear PD term plus a switching function about a sliding variable.

In this paper, we consider the problem of attitude control for flexible satellites based on the unit quaternion representation. It is assumed that the modal variables describing flexible elements are not measurable. For such a class of flexible satellites, a dynamic controller is given to achieve stability for the closed-loop system. The designed controller has two features. One is that it is in the form of an observer-based state feedback. The other is that the angular velocity feedback is not used.

2. Motion equations of a flexible satellite and problem formulation

In this section, the mathematical model of a flexible satellite is given. We adopt the unit quaternion to describe the attitude of a satellite. The associated quaternion is given by

$$\boldsymbol{q} = \begin{bmatrix} q_0 \\ \boldsymbol{q}_\mathrm{v} \end{bmatrix} \tag{1}$$

with

$$q_0 = \cos(\Phi/2), \quad \boldsymbol{q}_\mathrm{v} = \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} = \epsilon \sin(\Phi/2) \tag{2}$$

where ϵ is the unit Euler axis, and Φ is the rotation angel about the Euler axis. The quaternion components are not independent on each other, and they satisfy a single constraint as

$$q_0^2 + \boldsymbol{q}_\mathrm{v}^\mathrm{T}\boldsymbol{q}_\mathrm{v} = 1$$

For the quaternion $\boldsymbol{q}$ in Eq. (1), define the following matrix $\boldsymbol{\Xi}$:

$$\boldsymbol{\Xi}(q_0, \boldsymbol{q}_\mathrm{v}) = \left[-\boldsymbol{q}_\mathrm{v}, q_0 I - \boldsymbol{q}_\mathrm{v}^\times\right] \tag{3}$$

where $\boldsymbol{q}_\mathrm{v}^\times$ is the cross-product matrix defined by

$$\boldsymbol{q}_\mathrm{v}^\times = \begin{bmatrix} 0 & -q_3 & q_2 \\ q_3 & 0 & -q_1 \\ -q_2 & q_1 & 0 \end{bmatrix} \tag{4}$$

With the preceding notation, the quaternion kinematics equation is given as [6]

$$\dot{\boldsymbol{q}} = \begin{bmatrix} \dot{q}_0 \\ \dot{\boldsymbol{q}}_\mathrm{v} \end{bmatrix} = \frac{1}{2}\boldsymbol{\Xi}^\mathrm{T}(q_0, \boldsymbol{q}_\mathrm{v})\boldsymbol{\omega} \tag{5}$$

where ω is the satellite angular velocity. Due to the property of the matrix $\boldsymbol{\Xi}$, there holds

$$\boldsymbol{\omega} = 2\boldsymbol{\Xi}(q_0, \boldsymbol{q}_\mathrm{v}) \begin{bmatrix} \dot{q}_0 \\ \dot{\boldsymbol{q}}_\mathrm{v} \end{bmatrix} \tag{6}$$

Under the hypothesis of small deformations, by using the Euler theorem, the dynamic equations of a flexible satellite can be given by

$$\boldsymbol{J}\dot{\boldsymbol{\omega}} + \boldsymbol{\delta}^\mathrm{T}\ddot{\boldsymbol{\eta}} = -\boldsymbol{\omega} \times (\boldsymbol{J}\boldsymbol{\omega} + \boldsymbol{\delta}^\mathrm{T}\dot{\boldsymbol{\eta}}) + \boldsymbol{u} \tag{7}$$

$$\ddot{\boldsymbol{\eta}} + \boldsymbol{C}\dot{\boldsymbol{\eta}} + \boldsymbol{K}\boldsymbol{\eta} = -\boldsymbol{\delta}\dot{\boldsymbol{\omega}} \tag{8}$$

where $\boldsymbol{J}$ is the total inertia matrix which is symmetric, $\boldsymbol{u}$ is the external torque acting on the main body of the satellite, and $\boldsymbol{\eta}$ is the modal coordinate vector. In the modal Eq. (8), $\boldsymbol{C}$ and $\boldsymbol{K}$ are respectively the damping matrix and the stiffness matrix, which are in the following forms:

$$\begin{aligned} \boldsymbol{C} &= \mathrm{diag}\{2\zeta_1\omega_{\mathrm{n}1}, 2\zeta_2\omega_{\mathrm{n}2}, \ldots, 2\zeta_N\omega_{\mathrm{n}N}\} \\ \boldsymbol{K} &= \mathrm{diag}\{\omega_{\mathrm{n}1}^2, \omega_{\mathrm{n}2}^2, \ldots, \omega_{\mathrm{n}N}^2\} \end{aligned}$$

δ is the coupling matrix between flexible and rigid dynamics. In this paper, N elastic modes are considered. The corresponding natural frequencies are $\omega_{\mathrm{n}i}$, $i = 1, 2, \ldots, N$, and the associated dampings are ζ_i, $i = 1, 2, \ldots, N$. From Eqs. (7) and (8), the following dynamic equations of the flexible satellite can be obtained[15]:

$$\begin{cases} \dot{\boldsymbol{\omega}} = \boldsymbol{J}_\mathrm{mb}^{-1}[-\boldsymbol{\omega} \times (\boldsymbol{J}_\mathrm{mb}\boldsymbol{\omega} + \boldsymbol{\delta}^\mathrm{T}\boldsymbol{\psi}) + \boldsymbol{\delta}^\mathrm{T}(\boldsymbol{C}\boldsymbol{\psi} + \boldsymbol{K}\boldsymbol{\eta} - \boldsymbol{C}\boldsymbol{\delta}\boldsymbol{\omega}) + \boldsymbol{u}] \\ \dot{\boldsymbol{\eta}} = \boldsymbol{\psi} - \boldsymbol{\delta}\boldsymbol{\omega} \\ \dot{\boldsymbol{\psi}} = -(\boldsymbol{C}\boldsymbol{\psi} + \boldsymbol{K}\boldsymbol{\eta} - \boldsymbol{C}\boldsymbol{\delta}\boldsymbol{\omega}) \end{cases} \tag{9}$$

In dynamic Eq. (9),

$$\boldsymbol{J}_\mathrm{mb} = \boldsymbol{J} - \boldsymbol{\delta}^\mathrm{T}\boldsymbol{\delta}$$

is the main body inertia matrix and