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Satellite proximate interception vector guidance based on differential games

Dong YE [a], **Mingming SHI** [b,*], **Zhaowei SUN** [a]

[a] *Research Center of Satellite Technology, Harbin Institute of Technology, Harbin Institute of Technology, Harbin 150001, China*
[b] *Faculty of Science and Engineering, University of Groningen, Groningen 9747AG, The Netherlands*

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Abstract This paper studies the proximate satellite interception guidance strategies where both the interceptor and target can perform orbital maneuvers with magnitude limited thrusts. This problem is regarded as a pursuit-evasion game since satellites in both sides will try their best to capture or escape. In this game, the distance of these two players is small enough so that the highly nonlinear earth-centered gravitational dynamics can be reduced to the linear Clohessy-Wiltshire (CW) equations. The system is then simplified by introducing the zero effort miss variables. Saddle solution is formulated for the pursuit-evasion game and time-to-go is estimated similarly as that for the exo-atmospheric interception. Then a vector guidance is derived to ensure that the interception can be achieved in the optimal time. The proposed guidance law is validated by numerical simulations.

1. Introduction

Satellites can be a tool to intercept the opponent's critical satellite which serves in the space above the important field. In the satellite attacking-defense system, the attacking satellites often keep dormant on their hiding orbits. They will be revoked to perform orbital maneuvers and intercept the dangerous targets by the ground facilities or other early warning satellites. This interception problem is considered to enter the final phase when the attacking and escaping satellites move close enough so that the interceptor can identify the target with onboard electronic devices.

Massive papers have studied the control strategies for satellite interception when the target has no maneuverability. Based on the Clohessy-Wiltshire (CW) equation, Ichikawa and Ichimura[1] decomposed the satellite relative motion as the orbital planar motion and the motion outside orbital plane. The authors employed the fuel cost as the optimal objective and obtained a relative orbital control strategy, with three in-plane and one out-plane impulsive maneuvers. It is easy to design or operate proximate orbit rendezvous or interception by impulsive method. However, the precision of impulsive guidance often cannot satisfy the mission requirement since it is an open-loop control method. As for continuous thrust interception, the miss distance of variable thrust control method can be reduced with various control strategies. Lu

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* Corresponding author.
E-mail address: M.Shi@rug.nl (M. SHI).

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and Xu[2] studied the continuous satellite rendezvous problem for elliptical target's orbits, in which the thrust magnitude is limited. In this paper, an adaptive control strategy is proposed to overcome the difficulty brought by non-communication between the rendezvous satellites. Based on the output feedback control, Singla et al.[3] designed a structured model reference adaptive controller to solve the automated orbital rendezvous problem with measurement uncertainties. In Ref.[4], two Optimal Terminal Guidance (OTG) laws are developed for the exo-atmospheric interception with final velocity vector constraints. To make the problem solvable, a linear model is used to approximate the gravity difference between the target and the interceptor. The proposed guidance consumes much less fuel and requires a light computational load. Even the research results on non-maneuvering target interception or rendezvous have been applied in the real engineering, a more rigorous situation is that not only the interceptor can move toward the target, but the latter can perform orbital maneuver when it carries thrusters. Obviously, the interception will fail if the target can move in an impulsive way. Hence, continuous thrust is often presumed to make the problem sensible.

Traditionally, this problem is regarded as the non-cooperative rendezvous. Two solving approaches have been proposed: (A) robust sliding mode controller; (B) robust H_∞ controller. For the former, the readers can refer to Ref.[5], where Wu et al. developed a finite time observer and controller for the satellite interception with maneuverable target based on the non-singular terminal sliding mode theory. The method can make the position and velocity differences between the tracking and target satellites below an expected value. In Ref.[6], the authors studied relative motion control of spacecraft rendezvous on low elliptical orbit. To cope with the J_2 perturbation, atmospheric drag and thrust failure, the authors developed two robust controllers based on the optimal sliding mode control and back stepping sliding control. For the latter, Gao et al.[7] developed a robust H_1 state feedback controller to solve the satellite rendezvous problem with parametric uncertainties, system disturbances and input constraints. Based on the Lyapunov analysis, a set of Linear Matrix Inequalities (LMIs) were obtained under multi-objective requirements. In Ref.[8], Deng et al. studied the finite time satellite interception orbital control problem. A state feedback controller was designed by considering parametric uncertainties, finite time performance, control input constraints and pole assignment requirements. LMIs were used to solve the finite time controller. Simulations showed that the system was asymptotically stable and the requirements for system performance, input constraints and pole assignment were all satisfied.

Recently, another method is developed from the differential games theory which regards the interception problem where both sides have maneuvering capabilities as a pursuit-evasion game. Isaacs firstly concentrated on this problem and defined the two-side optimal solution as the saddle solution.[9] With quadratic objective functions, Menon and Calisa[10,11] obtained a feedback control strategy for spacecraft interception with saturated control input by the back stepping method. In Ref.[12], a near-optimal feedback control for minimax-range pursuit-evasion problems between two constant-thrust spacecraft was generated by periodically solving the differential game problems with a modified first-order differential dynamic programming algorithm after the system state was updated. This new technique only requires a rough estimation of the optimal control to start the solving algorithm, instead of the accurate solution of a complete two-point boundary value problem, and hence can be implemented in the real time more easily. However, these papers assumed that the satellites have great maneuver capability, which is impossible in the real engineering. For nonlinear dynamics, the analytic solution for the two-person zero-sum differential games is often difficult to solve for the extremely complicated form of the Hamilton–Jacobi-Isaacs (HJI) partial differential equations. Hence, most literature dedicated to finding the open-loop saddle-point solution. Pontani and Conway[13,14] gave a numerical method to solve the open-loop trajectory of the three-dimensional satellite pursuit-evasion interception, where each spacecraft had a modest capability to maneuver. In the interception, the objective of the pursuer was to minimize the elapsed time after which it hit the target satellite, whereas the evader tried to postpone that instant as late as possible. A pre-solution of the saddle-point equilibrium was firstly derived by genetic algorithms. Then this solution was regarded as the initial guess and substituted into the semi-analytic method to find the accurate pursuit-evasion trajectory. The intensive random search and collocation method in Ref.[13] offers the possibility of searching a global optimal solution for the complex nonlinear pursuit-evasion games. However, it occupied high computational resources. In Ref.[15], the authors applied sensitivity methods to the orbital pursuit-evasion problem in the same scenario as Ref.[13], which sharply reduced the computation burden for the numerical solving of nonlinear satellite pursuit-evasion trajectories. This makes the real time satellite interception possible.

Compared with numerical solving open-loop trajectory, it is more difficult to derive the closed-loop control. Ghosh and Conway[16] presented an extremal-field approach to synthesize nearly-optimal feedback controllers for the non-linear two-player pursuit-evasion games. The proposed method utilized the universal Kriging technique to construct the surrogate model of the feedback controller, which was capable of generating the sub-optimal control based on current state information. In this method, the open-loop extremals were first generated offline by a direct or indirect method, and then the real time feedback map was obtained by interpolating the controls of these open-loop extremals. With the same method, Stupik et al.[17] studied the satellite combat based on the linearized CW equation. The sub-optimal feedback solution was interpolated by the standard solutions which were pre-calculated with various initial conditions. Since the dynamics is reduced, the number of conjugates that needs to solve decreased from 12 in Ref.[13] to 3. This sharply improved the open-loop extremal's offline pre-solving ability. However, the method is derived for solving the open solution. Although the authors employed Kriging technique to construct a real-time feedback control, it still cannot guarantee the optimality of the solution and successful interception of agile satellite. Jagat and Sinclair[18] formulated the linear spacecraft pursuit-evasion interception as a two-player zero-sum differential game. A finite horizon linear control law was obtained by applying the Linear-Quadratic (LQ) differential game theory. Then a nonlinear control law was obtained by solving the state-dependent Riccati equation method. The results are not practical since in the real situation the evader will adopt the control which can make it escape away as soon as possible. Tartaglia and